ENEE324. Problem set 6

Date due: November 26, 2014

1. (The Chernov bound, pp.284-286).

(a) Let $X \sim \mathcal{N}(0,1)$. Use the Chernoff bound to show that $P(X \ge c) \le e^{-c^2/2}$. Then take $c = 2$ and compare the this estimate with the true value (please give the numbers for both ways of finding $P(X \ge 2)$).

(b) Now let $X \sim \text{Poisson}(\lambda)$ (p.78 in the textbook). Use the Chernoff bound to estimate $P(X \ge c)$. For which c does your calculation produce a trivial upper bound $P(X \ge c) \le 1$?

2. A fair coin is tossed 6 times, yielding a sequence of Heads and Tails. Let X be the parity of the number of Heads (the remainder of the division of the number of heads by 2) and let Y be the remainder of the division of the number of Tails by 3.

(a) Find the PMFs $p_X(k)$, $p_Y(k)$.

(b) Are X and Y correlated? Are they independent?

3. Consider two continuous r.v.'s X and Y that are uniformly distributed in the region $\{|x+y| \le 1, |x-y| \le 1/2\}$ of the (x,y) plane.

(a) Find $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$ for all x and y.

(b) Find $\mathbf{E}[X|Y], \mathbf{E}[X]$.

4. Packets arrive at the server with an exponential wait time, so that the kth packet arrives after $X_1 + X_2 + \cdots + X_k$ minutes, where $X_k, k = 1, 2, \ldots$ are exponential random variables with $EX_k = 2$ for all k. Find the probability P that the third packet arrives no earlier than 20 minutes after the start of the experiment. Estimate P using the Markov inequality.

5. Let $X_1, X_2, \ldots,$ be i.i.d. RVs with $EX = 2, \text{Var}(X) = 9$. Define $Y_i = X_i/2^i$ for all i. Finally let $T_n = \sum_{i=1}^n Y_i$ and $A_n = T_n/n$.

(a) Find E and Var for Y_n, T_n, A_n.

(b) Do the RVs Y_n, T_n, A_n converge in probability to a number, and if yes, what are the limiting values?