

ENEE324-03. Problem set 7

Date due: Tuesday April 14, 2015

1. Let X be a continuous RV with PDF $f_X(x) = 4x^3$ for $0 < x < 1$ and zero otherwise. Find the PDF of the RV $Y = 1 - 3X^2$.

2. The time to failure of a computer screen equals $10,000X$ hrs, where X is an RV with PDF

$$f_X(x) = \begin{cases} 2/x^2 & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What percentage of monitors lasts fewer than 15,000 hrs?
 (b) What percentage of the monitors that last fewer than 15,000 hrs, lasts between 10,000 and 12,500 hrs?

3. Let $X \sim \mathcal{N}(0, 1)$. Find $E(e^{aX})$, where a is a fixed real number. Find the PDF of $Y = X^2$.

4. Let X be an RV with PDF

$$f_X(x) = \begin{cases} ax & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

Let B be the event $\{|X - 2.2| \leq 0.5\}$. Find $EX, P(B), E[X|B]$ (note that none of the answers should contain a)

5. Let $X \sim \text{unif}[0, 1]$ and $Y \sim \exp(2)$ be independent RVs. Let $Z = X + Y$. Find $P(Y \geq X), f_{Z|Y}(z|y), f_{Y|Z}(y|z)$ (note that the values of the PDFs should be specified for all values of the RVs from ∞ to ∞).

6. Consider two independent exponential RVs $X \sim \exp(\lambda_1)$ and $Y \sim \exp(\lambda_2)$. Find $P(X + Y \geq 2), P(\max(X, Y) \geq 5)$.

7. Let $X \sim \text{Laplace}(2)$. Let $Z = X + Y$ where $Y \sim \text{Unif}[0, 2]$.

(a) Compute $\mathbf{P}(Z > 2|Y|)$.
 (b) Find the PDF $f_Z(z)$.