ENEE324 Fall 2016. Problem set 7

Date due November 9, 2016

Please justify your answers

1. Roll a 6-sided die n times and denote by X and Y the random numbers of 2's and 3's obtained in the n rolls. Find the covariance of X and Y.

2. Let $X \sim \text{unif}\,[0,1]$ and $Y = X^2$. Find $\rho(X,Y)$.

3. Suppose that an urn contains B balls of which r are red and $b = B - r$ are blue. Balls are taken at random one by one. If a red ball is drawn, we remove it and put back one blue ball instead of it; if a blue ball is drawn, we simply put it back into the urn. Let X_n be the number of blue balls after n repetitions. Find EX_n.

(Hint: (a) let $X_i, i = 0, 1, \dots, n$ be the number of blue balls in the urn after i trials; show that $E(X_i|X_{i-1}) = 1 + (1 - \frac{1}{B})X_{i-1}$, then compute the expectation of the last relation to find an equation that connects EX_i and EX_{i-1}; then show that $EX_n = B - r(1 - \frac{1}{B})^n$.)

Useful formulas:

Bivariate normal pdf with $\mu_X = \mu_Y = 0$ was considered in class (p.254). A bivariate normal with nonzero means has the form

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\Big\{ -\frac{\big(\frac{x-\mu_X}{\sigma_X}\big)^2 - 2\rho\frac{x-\mu_X}{\sigma_X}\frac{y-\mu_Y}{\sigma_Y} + \big(\frac{y-\mu_Y}{\sigma_Y}\big)^2}{2(1-\rho^2)} \Big\}, \ -\infty < x, y < \infty$$

$$f_{Y|X}(y|x) = \frac{1}{\sigma_Y\sqrt{2\pi}\sqrt{1-\rho^2}} \exp\Big\{ -\frac{(y - \mu_Y - \rho(\sigma_Y/\sigma_X)(x-\mu_X))^2}{2\sigma_Y^2(1-\rho^2)} \Big\}, \ -\infty < y < \infty.$$

4. The joint PDF of the RVs X and Y is bivariate normal with the averages $\mu_X = 3, \mu_Y = 2.5$, standard deviations $\sigma_X = 0.5, \sigma_Y = 0.4$ and the correlation coefficient $\rho(X,Y) = 0.4$. Find $P(Y \ge 3|X = 3.6)$.

5. Given two RVs X, Y whose joint PDF is bivariate normal, we also know that $\sigma_x = \sigma_Y$. Are the RVs $U = X + Y, V = X - Y$ independent?

(hint: find the joint PDF of U, V to be bivariate normal, find $\rho(U,V)$. Note that there is no need to find explicitly $f_{UV}(u,v)$ to compute $\rho(U,V)$.)

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6. (a) Suppose that the transform of an RV X is $M_X(s) = \frac{1}{10}e^s + \frac{3}{10}e^{3s} + \frac{5}{10}e^{4s} + \frac{1}{10}e^{7s}$. Find $P(X \le 5)$.

(b) The transform of an RV X is $M_X(s) = e^{2s^2}$. Find $P(0 < X < 1)$.