ENEE324. Problem set 7

Date due April 13, 2016

Explanations are required, no credit for just the answer.

1. Let X be an RV supported on $(0,\infty)$ with pdf $f_X(x) = xe^{-x}$. Suppose that Y is another RV such that for any given $x > 0$, the conditional distribution $Y|X = x$ is uniform on the segment $(0, x]$.

(a) Find $f_{XY}(x, y)$; (b) Find $EX, \mathrm{Var}(X)$; (c) Find $EY, \mathrm{Var}(Y)$; (d) Compute the transform $M_X(s)$ and find $E(X^n)$ using $M_X(s)$; (e) Find the covariance $\mathrm{Cov}(X, Y)$ and the correlation coefficient ρ_{XY}.

2. (a) Consider two independent identically distributed binomial random variables $X, Y : X \sim \mathrm{Binom}(n, p); Y \sim \mathrm{Binom}(n, p)$ and let $Z = X + Y$. Compute the transform $M_Z(s)$. What is the distribution of Z?

(b) Now solve question (a) with $X \sim \mathrm{Poisson}(\lambda_1)$ and $Y \sim \mathrm{Poisson}(\lambda_2)$. Using the obtained distribution of Z, find $p_{X|Z}(x|z)$. Explain that this (conditional) pmf is binomial, identify the parameters of the binomial distribution.

3. Two buses independently arrive at a bus stop, and the arrival time of each of them is uniformly distributed between 10 am and 12pm. Find the probability that the gap between their arrivals is no more than 15 minutes.

4. A (mixed) RV X has CDF

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 0.1 + 0.15x, & 0 \le x \le 6 \\ 1, & x \ge 6. \end{cases}$$

(a) Make a sketch of $F_X(x)$, find the pdf $f_X(x)$.

(b) Let $Y = X^2$. Find $F_Y(x), f_Y(x), x \in \mathbb{R}$; find EY.

5. A random point X is chosen on the segment $[1, 2]$ with uniform distribution. After that, a random point Y is chosen on the segment $[X, 2]$, also with uniform distribution.

(a) Find $f_{Y|X}(y|x), f_{XY}(x, y), f_Y(y)$. Compute EY using $f_Y(y)$ and the definition of mathematical expectation.

(b) Use the law of iterated expectations to compute EY in a different way than in (a); check that you get the same answer.

(c) If X and Y are random sides of a rectangle in the (x, y) plane, what's the expected area of the rectangle?