ENEE324. Problem set 7

Date of issue: Dec. 1, 2014; due on Dec. 10, 2014.

1. Consider a Poisson process with average time between arrivals 10s. Let X_1 be the random time to the first arrival from an arbitrary time, and let X_2 be the time to the second arrival counting from the time of the first arrival.

(a) Find $f_{X_1}(x), f_{X_2}(x)$.

(b) Find $f_{X_1,X_2}(x, y)$.

(c) Find $\mathbf{P}(X_1 + X_2 \le 25s)$.

(d) Find $\mathbf{P}(X_2 - X_1 \le 15s)$.

2. You need to enter traffic on a road. The average distance between cars on your side is 60 ft. and traffic is moving at 30mph. Assume that all cars are moving at the same speed but that distances are distributed exponentially.

(a) Find the PDF of the random time between cars.

(b) You need a 2.5 second break in traffic to safely enter it. What is the probability that you will be able to enter without waiting for a break in traffic?

(c) Traffic on the other side of the road has the same characteristics. To cross the road on foot you need 2s. What is the probability that you will be able to cross without waiting for a break in traffic?

3. Consider a Bernoulli process with probability $\mathbf{P}(X_i = 1) = 1/3, i = 1, 2, \dots$.

(a) Find the probability of 4 arrivals within slots $1, 2, 3, \dots, 10$.

(b) Find the probability of 3 arrivals among slots $1, 3, 4, 9, 10, 11$.

(c) What is the probability of the series $\mathbf{P}(X_{i-2} = 1, X_{i-1} = 1, X_i = 0, X_{i+1} = 0)$, for some given i ?

(d) What is the PMF and the expected time to the third arrival?

(e) What is the probability that the third arrival occurs in slot 7?

4. Consider a Markov chain given by the matrix

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/4 & 1/4 \\ 0 & 1/4 & 1/4 & 1/2 \end{bmatrix}$$

Draw a state transition diagram, classify the states of the chain. Does the chain have recurrent classes? Find the stationary distribution of the chain.

5. A wheel installed on a tabletop is divided into 3 equal sectors, colored 1,2, and 3. A mark on the tabletop points to a point on the circumference of the wheel. The wheel is repeatedly sent into a free spin until it comes to a stop. Consider a stochastic process $X_0, X_1, X_1, \dots$ where X_i is an r.v. that represents the color of the sector chosen as a result of the ith spin. Consider the process $Y = (Y_1, Y_2, \dots,)$ where $Y_i = (X_{i-1}, X_i), i = 1, 2, 3, \dots$.

(a) Set up a Markov chain describing Y. Classify its states and recurrent classes.

(b) If the chain has a stationary distribution, compute it.