ENEE324-03. Problem set 8

Date due: Tuesday April 21, 2015

1. There are n boxes. I keep throwing marbles into the boxes, and each toss lands the marble into one of them with equal probability $1/n$. What is the expected number of tosses before there are no more empty boxes left?

2. Let X and Y be jointly distributed with $\rho(X,Y) = 2/3, \sigma_X = 1, \text{Var}(Y) = 9$. Find $\text{Var}(3X - 5Y + 7)$.

3. We are transmitting text over a communication link, sending each letter as some sequence of bits, i.e., 0s and 1s (as in the Morse code). Suppose that the number of bits per letter is a geometric RV with the parameter p. Suppose that we transmit at the rate of 1000bits/sec. What is the distribution of the RV T, the time it takes to send one letter of the text.

4. A random point X is chosen on the segment $[0,1]$ with uniform distribuion. After that, a random point Y is chosen on the segment $[0,X]$, also with uniform distribution.

(a) Find $f_{Y|X}(y|x), f_{XY}(x,y), f_Y(y)$ (hint: compute the functions in this order). Compute EY using $f_Y(y)$ and the definition of mathematical expectation.

(b) Use the law of iterated expectations to compute EY in a different way than in (a); check that you get the same answer.

(c) If X and Y are random sides of a rectangle in the (x,y) plane, what's the expected area of the rectangle?

5. Let $X \sim \text{unif}\,[0,1]$ and let $a > 0, b > 0$. Find the transform $M_Y(s)$, where $Y = aX + b$ and show that $Y \sim \text{unif}\,[b, a+b]$.

6. For an RV X, $M_X(s) = (e^s + 2)^4/81$. Find $P(X \le 2)$.