ENEE324. Problem set 8

Date due April 27, 2016

Explanations are required, no credit for just the answer.

1. Consider a pair of jointly Gaussian RVs X and Y which are not assumed to be uncorrelated. Assuming that $\sigma_X = \sigma_Y$, argue that the RVs $V = X + Y$ and $W = X - Y$ are independent.

2. Two 6-sided dice D_1 and D_2 are rolled. Let $X = D_1 + D_2$ and $Y = |D_1 - D_2|$. What is the covariance of X and Y? Are X and Y independent?

3. Consider a sequence of iid RVs $X_i, i = 1, 2, \ldots$, with pdf $f(x) = \begin{cases} 4x(1-x) & 0 \le x \le 1 \\ 0 & \text{o/w} \end{cases}$. Let $S_n = (X_1 + X_2 + \cdots + X_n)/n$. Find $\lim_{n \to \infty} S_n$.

4. We are estimating the mean of the distribution of an RV X by computing the sample mean of n outcomes, aiming at the estimation error no more than $2\sigma_X$. How large should n be to achieve the goal with probability at least 98%?

5. (CLT) Suppose X is an RV with PDF $f_X(x) = 0$ if $x \le 1$ or $x \ge 3$, and

$$f_X(x) = \frac{1}{9}(x + 5/2) \text{ o/w.}$$

Let $\bar{X}$ be the average of $n = 24$ sapmples. Find the approximate value of the probability $P(2 < \bar{X} < 2.5)$.

6. Outcomes of an experiment are random numbers with mean 0 and variance 10. Find an upper bound on the probability that a particular outcome is 40 or more.