ENEE324-03. Problem set 8

1. Consider a Markov chain given by the matrix

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/4 & 1/4 \\ 0 & 1/4 & 1/4 & 1/2 \end{bmatrix}$$

Draw a state transition diagram, classify the states of the chain. Does the chain have recurrent classes? Find the stationary distribution of the chain.

2. Consider a Bernoulli process $X_0, X_1, X_2, X_3, \ldots$, where for every i, the r.v. $X_i \sim$ Bernoulli($1/2$). Let $Y_k = 1/2(X_k + X_{k-1})$. Do the r.v.'s $Y_k, k = 1, 2, 3, \ldots$ form a Markov chain?

3. A wheel installed on a tabletop is divided into 3 equal sectors, colored 1,2, and 3. A mark on the tabletop points to a point on the circumference of the wheel. The wheel is repeatedly sent into a free spin until it comes to a stop. Consider a stochastic process $X_0, X_1, X_1, \ldots$ where X_i is an r.v. that represents the color of the sector chosen as a result of the ith spin. Consider the process $Y = (Y_1, Y_2, \ldots,)$ where $Y_i = (X_{i-1}, X_i), i = 1, 2, 3, \ldots$.

(a) Set up a Markov chain describing Y. Classify its states and recurrent classes.

(b) If the chain has a stationary distribution, compute it.