ENEE324-03. Problem set 9

Date due: Tuesday May 5, 2015

1. Let X, Y be two RVs such that

$$f_{XY}(x,y) = ye^{-y(1+x)},\ x > 0; y > 0.$$

Show that $E(X)$ does not exist. Find $E(X|Y)$.

2. A safety net can stand the fall of the total of 2700 pounds of objects from a construction site. Say 12 random objects are falling down into the net at the same time, and the weight of each of them is $\sim \mathcal{N}(225, 625)$. What is the probability that the net withstands the impact?

3. Choose $n = 150$ random points x_i in the interval $(0, 1)$. Compute

$$P\Big(\Big|\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{2}\Big| \le 0.02\Big).$$

4. Let X be an RV with $EX = 42, \mathrm{Var}(X) = 60$. Take 25 realizations of X and let M be the mean value of the obtained sample. Is it true that $P(38 \le M \le 46) \ge 0.85$?

5. Let the joint PDF of X, Y be $f_{XY}(x, y) = e^{-x}, 0 < y < x < \infty$. Find the marginal PDFs of X and Y. Find the correlation coefficient of X and Y.

6. Packets arrive at the server according to a Poisson process with rate $\lambda = 2$. Suppose that within the time interval of length 4 sec. there were 6 arrivals. What is the probability that the 1st packet arrived with 1 sec. from the beginning of the interval?

Hint: Let $X_i, i = 1, 2, \ldots, 6$ be independent uniform RVs, and $X_i \sim \mathrm{Unif}[0, 4], i = 1, \ldots, 6$. Then the random time of the first arrival, conditional on the event $N(4) = 6$, is $Y = \min(X_1, X_2, \ldots, X_6)$.

7. Suppose that customers enter a store at a Poisson rate $\lambda = 0.5$ per hour. Calculate the following:

(1) the probability that 8 customers arrive over a three hour period
(2) the probability that 3 customers arrive in an hour given that none arrived the previous hour
(3) the probability that that 1 customer arrives in an hour and 2 customers arrive the next hour