ENEE324 Fall 2016. Problem set 9

Date due December 7, 2016

Please justify your answers

1. There are six points 1,2,3,4,5,6 placed on a circle. Consider a random walk that moves from a point to one of its two neighbors (clockwise or counterclockwise) with probability 1/2. Suppose we start at 1, what is the probability that in 4 steps the walk returns to 1? What is the probability that in 5 steps the walk lands either at 2 or at 6?

2. Given a Markov chain specified by the following matrix

$$P = \begin{pmatrix} 2/5 & 0 & 0 & 3/5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 1/3 & 0 & 2/3 \end{pmatrix}$$

Identify transient and recurrent states and recurrent classes.

3. Three players randomly draw cards from a deck of 52 cards successively and with replacement. Player 1 starts drawing cards until he gets an ace. After than player 2 draws cards until he gets a spade, after that player 3 draws cards until he gets a face card. After that the entire process repeats. What is the long-run proportion of cards drawn by each player?

4. Consider a Markov chain given by the following matrix:

$$P = \begin{pmatrix} 0.3 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.1 & 0.3 & 0.1 & 0 & 0.2 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Suppose we start in state 6. What is the probability that eventually the chain will be absorbed into state 4?

5. Consider a Markov chain given by the matrix

$$P = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.32 & 0.15 & 0.53 \\ 0.6 & 0.13 & 0.27 \end{pmatrix}$$

Suppose that the initial state of the chain is 1,2, or 3 with probability 1/3. Find the expected wait time from the empty chain to the first appearance of each of the three states.

6. I have r phone chargers which I use at home or in the office. With probability $1-p$ my phone's charge is low when I leave each of these two places, and I decide to take a charger with me from the location that I am leaving to where I am headed. It may be that there are no chargers at the place that I am leaving, so I go empty-handed. Define a Markov chain with $r+1$ states, where each state corresponds to the number of chargers at home and in the office (e.g., 3 and $r-3$).

(a) Find the limiting (steady-state) probabilities of the chain.

(b) What is the proportion of time that I cannot take along a charger when I need one?

(c) Say $r = 3$, what is the value of p such that the fraction of time found in (b) is the largest?