ENEE324. Problem set 9

Date due May 4, 2016

Explanations are required, no credit for just the answer.

1. Given that the number of arrivals by time t in a Poisson process with intensity (arrival rate) λ equals n.

(a) Find the distribution of the number of arrivals in an interval $[0, s]$ for a given $s < t$. (Hint: your task is to compute the conditional probability $P(N(s) = i|N(t) = n), 0 \le i \le n$, where $N(t)$ is the number of arrivals by time t).

(b) Now let X_k be the time of the kth arrival. Show that $E[X_k|N(t) = n] = (kt)/(n+1)$. (You may use the fact that $f_{X_k|N(t)}(x|n) = \binom{n}{k}\frac{k}{t}(\frac{x}{t})^{k-1}(1-\frac{x}{t})^{n-k}, 0 \le x \le t$.)

2. In a Poisson process with rate $\lambda = 3$ we have recorded 8 arrivals in time $[0, 5]$. What is the probability that the first arrival occurred in the time interval $[0, 2]$?

3. The matrix of transitional probabilities of a Markov chain has the form

$$P = \begin{pmatrix} 2/5 & 0 & 0 & 3/5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 1/3 & 0 & 2/3 \end{pmatrix}.$$

Specify the classes and determine which classes are transient or recurrent.

4. A skier goes down the slope repeatedly one time after another. He may fall or not, and if he falls once in the last two runs, then the run immediately after them will end in a fall with probability 1/2, if he falls in each of the last two runs, then he'll fall with probability 7/8, while if the last two runs went with no falls, then in the run immediately after them he will crash with probability 1/8. Last Sunday the fellow skied four times and crashed in the first two runs. What's the probability of the fall in the fourth run on that day? In the long run, what's the probability of two falls in a row?

5. Cards are drawn randomly,successively, and with replacement from a well-shuffled deck of 52 cards. There are 3 face cards and one ace in each of the 4 suits. The event A is that an ace is drawn before a face card.

(a) Find $P(A)$. For this, set up a Markov chain of 3 states $1, 2, 3$, where state 1 is that the card in the next step is neither a face card nor an ace, state 2 is that it's an ace, and state 3 is that it's a face card. Then find p_{12}^n for $n \ge 1$ and find the limit as $n \to \infty$.

(b) Find the expected wait for the event A to happen counting from the start of the experiment (see Sect.7.4 in the textbook).