ENEE324-03 Spring 2019. Problem set 9

Date due May 9, 2019

1. A six-sided fair die is rolled repeatedly. Denote by X_n the RV equal to the maximum of the first n rolls.

(a) Does the sequence $X_n, n = 1, 2, \ldots$ form a Markov chain? Why?

(b) If it does, calculate the transition probability matrix P, specify the closed classes, and determine which states are recurrent and which are transient.

2. Consider a Markov chain with states $S = \{1, 2, 3, 4, 5\}$ and the following matrix of transition probabilities:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.4 & 0.1 & 0 \\ 0 & 0.2 & 0.3 & 0 & 0.5 \\ 0 & 0 & 0 & 0.3 & 0.7 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

(a) Classify the states of the chain into transient and recurrent. Identify closed classes.

(b) Compute the expected time of entering one of the states 4 of 5 (does not matter which) starting from each of the transient states.

3. A and B are playing multiple rounds of a game. The probability for A to win a round equals p if he lost the previous round. The probability for A to lose a round equals p if he won the previous round.

(a) For $n > 1$ show that if A wins the first round, the probability for him to win the nth round is $\frac{1}{2} + \frac{1}{2}(1 - 2p)^n$.

(b) Find the expected number of rounds that A will play between two consecutive wins.

4. Customers arrive at a bank at a Poisson rate λ. Suppose that two customers arrived during the first hour. What is the probability that

(a) both arrived within the first 20 minutes?

(b) at least one arrived during the first 20 minutes?

5. A Poisson process with arrival rate λ per hour registered 6 arrivals in the first four hours. Conditional on that, what is the probability that the first arrival occurred during the first hour?