Mathematical Prerequisites for ENEE324

Students usually have difficulties with mathematical questions similar to those listed below. Check yourselves (the solutions will not be collected or graded).

You are supposed to be able to solve the following problems without any outside assistance. Do not use your calculator, do not use Matlab or Wolfram alpha. All of the concepts involved are explained in detail on wikipedia. Note that the below problems represent a very small part of basic calculus, and are listed here only because the skills defined by them are essential for the 324 class.

Solving these or similar problems will be a part of your tests, and mistakes made in these calculations will decrease your score and may affect your grade for this course.

1. Draw plots of the following functions (y is a function of x):

$$x+y=1;\ x-y=10;\ y=x(1-x);\ y=\sqrt{1-x^2}.$$

2. Draw the area of the (x,y) plane that satisfies

$$(a)\ |x-y|\le 1/2$$
$$(b)\ x^2+y^2\le 2$$
$$(c)\ 0\le x\le 1, 0\le y\le 1; \max(x,y)\le z,\ \text{where } z \text{ is a number between 0 and 1.}$$

3. Recall the definition of the limit of a sequence and of a function. Make sure you understand what we mean by saying that $\lim_{n\to\infty} a_n = 5, \lim_{n\to\infty} a_n = -\infty, \lim_{n\to\infty} a_n$ does not exist; $\lim_{x\to 0} f(x) = a$ (not the formal ϵ-δ statements, but the meaning of this concept, e.g., draw a small interval around 5, then for all $n \ge n_0$ the numbers a_n are all in that interval).

Without using the computer or the l'Hôpital rule, find

$$(a)\lim_{x\to 0}\frac{e^x-1}{x};\ (b)\lim_{x\to\infty}\left(1-\frac{1}{2nx}\right)^x, n>0;$$
$$(c)\lim_{x\to 0}\frac{\sin ax}{x};\ (d)\lim_{x\to 0}\frac{\ln(1+x)}{x};$$
$$(e)\lim_{x\to 0} x\ln x$$

4. Recall the geometric sum, geometric series, and the binomial formula (*I will assume that you remember them*). Evaluate the following sums in a closed form:

$$(a)\sum_{i=1}^{n} x^i, x\in\mathbb{R}; (b)\sum_{i=1}^{n} 2^{-2i};\ (c)\sum_{i=0}^{\infty} x^i, |x|<1$$
$$(d)\sum_{i=0}^{n}\frac{\lambda_1^i\lambda_2^{n-i}}{i!(n-i)!};\ (e)\sum_{n=2}^{\infty}(1/3)^n;\ (f)\sum_{k=1}^{\infty}0.8^{k-1}.$$

5. (a) Compute the areas of the regions $(a)x+y\le\frac{1}{2}; (b)x-y\le\frac{1}{2}; (c)xy\le 1/4; (d)x^2+y^2\le 1$ inside the square $0\le x,y\le 1$. First find the area by drawing a picture and computing directly the areas of these regions. Nest write an integral that computes the same area and evaluate it. Perform the calculation in two ways, (1) integrating first on x then on y, and then (2) integrating first on y

and then on x. Make sure that the answers obtained in 3 different ways match. Do not stop until you understand all the details of these calculations.

(b) Compute

$$\int_0^3 x\ln x dx; \int_{x:0\le\tan x\le y} dx;$$
$$\int_0^\infty\int_0^\infty xye^{-\frac{1}{2}(x+y)}dxdy; \qquad \int_0^\infty\int_{2y}^\infty xye^{-x^2-y^2}dxdy.$$

6. Let $D=\{(x,y): 1\ge x\ge y\ge 0\}$. Compute $\iint_D xydxdy$.

7. (a) Find x_1, x_2, x_3 given that

$$\begin{cases} x_1 = 0.4x_1 + 0.3x_2 \\ x_3 = 0.2x_2 + 0.8x_3 \\ 1 = x_1 + x_2 + x_3 \end{cases}$$

(b) Solve the system of linear equations $\pi = \pi P$, where π is a vector $(\pi_i, i=1,\dots,4)$ and

$$P = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 \end{pmatrix}$$

8. Write the following integral in polar coordinates:

$$\iint_{-\infty}^{+\infty} e^{-\frac{x^2+y^2}{2}}dxdy$$

9. You must understand the little-o notation, e.g., $f(x)=o(x), x\to 0, f(x)=o(1)$. If not, please read a calculus textbook. For $x\to 0$ show that $(1+x)^n = 1+nx+o(x)$. Show that $x^\alpha \ln x = o(1), \alpha>0, x\to 0$.

10. What is the power series for e^x around $x=0$? Around $x=-2$? What is the power series for $(1+x)e^x$ around $x=0$? For $\ln(1+x)$ around $x=2$?

Evaluate the following sums in a closed form:

$$\sum_{n=0}^\infty e^{-a}\frac{x^n}{n!}; \quad \sum_{n=1}^\infty (-1)^{n-1}\frac{x^n}{n};$$

11. Evaluate $f'(x)$ if $f(x)$ is given by $(a)\, x^2e^{-x}$, $(b)\, x^{-1/2}$, $(c)\, \frac{1}{\sqrt[3]{x}}$, $(d)\, \tan(x)$, $(e)\, \arctan(x)$, $(f)\, \arcsin(x)$, $(g)\, \frac{\log_2(x)}{x^2}$ etc. You are supposed to be able to compute these expressions without any computer-like help or reference materials.

12. Compute $\frac{\partial^2}{\partial x\partial y}((1-e^{-x^2})(1-e^{-y^2}))$. Compute $\frac{d}{dx^n}(\frac{\lambda}{\lambda-x})$ for all natural numbers n.