Homework 10 Solutions

29.38 a) Current density from the displacement current

$$\oint \vec{B}\cdot d\vec{l} = \cancel{\mu_0 I} + \mu_0\epsilon_0 \frac{d}{dt}\int_A \vec{E}\cdot d\vec{A}$$

$$j_D = \epsilon_0 \frac{d}{dt} E = \epsilon_0 \frac{d}{dt}\frac{Q}{A\epsilon_0} = \frac{I}{A}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$j_D = \frac{0.52A}{\pi\, 16\times10^{-4}\,m^2}$$

$$= \frac{0.52}{\pi(.16)}\times10^{2}\,\frac{A}{m^2} = 1.03\times10^{2}\,\frac{A}{m^2}$$

b) $$\frac{dE}{dt} = \frac{1}{A\epsilon_0}\frac{dQ}{dt} = \frac{I}{A\epsilon_0} = j_D\frac{1}{\epsilon_0}$$

$$= \cancel{0.52A} \quad \frac{1.03\times10^{2}A\ Nm^2}{m^2\ 8.85\times10^{-12}C^2}$$

$$= \frac{1.03}{.885}\times10^{13}\,\frac{N}{Cs} = 1.17\times10^{13}\,\frac{N}{Cs}$$

c) What is B at $r = 2\times10^{-2}\,m$

$$B\,2\pi r = \mu_0 \pi r^2 j_D$$

$$B = \frac{\mu_0 r}{2} j_D = 4\pi\times10^{-7}\frac{Tm}{A}\ \frac{2\times10^{-2}m}{2}\ 1.03\times10^{2}\frac{A}{m^2}$$

$$= 0.4\pi\, 1.03\times10^{-6}\,T = 1.3\times10^{-6}\,T$$

d) At $r = 1.0\,cm$? $\quad B = \frac{1}{2}\,1.3\times10^{-6}\,T = 6.5\times10^{-7}\,T$

29.40 $I_c = 1.8\,mA = 1.8 \times 10^{-3} A$, $d = 2 \times 10^{-3} m$

a) At $t = 0.5 \mu s$, find Q, E, V.

$$Q = \cancel{1.8 \times 10^{-3}}\ I_c t$$

$$= 1.8 \times 10^{-3} A \ 0.5 \times 10^{-6} s$$

$$Q = 9 \times 10^{-10} C$$

$$E = \frac{Q}{A\epsilon_0} = \frac{9 \times 10^{-10} C \qquad N m^2}{5 \times 10^{-4} m^2 \ 8.85 \times 10^{-12} C^2}$$

$$= \frac{9}{8.85(.5)} \times 10^5 \frac{N}{C} = 2.0 \times 10^5 \frac{N}{C}$$

$$V = Ed = 2.0 \times 10^5 \frac{N}{C} \ 2 \times 10^{-3} m$$

$$= 4 \times 10^2 \frac{J}{C}$$

b) $$\frac{dE}{dt} = \frac{E}{t} = \frac{2 \times 10^5 N}{0.5 \times 10^{-6} s\, C} = 4 \times 10^{11} \frac{N}{C s}$$

$\frac{dE}{dt}$ is constant since I_c is constant

c) $$j_D = \frac{I_c}{A} = \frac{1.8 \times 10^{-3} A}{5 \times 10^{-4} m^2}$$

$$= \frac{1.8}{.5} \cancel{\times 10} \frac{A}{m^2} = 3.6 \frac{A}{m^2}$$

$$I_D = I_c = 1.8 \times 10^{-3} A$$

32.2 $\vec{S} = \frac{1}{\mu_0} E \times B$

$\Rightarrow$ $\vec{S}$ gives the direction of wave propagation

a) E along x, B along y, $S \sim \hat{i} \times \hat{j} = \hat{k}$

$\Rightarrow$ z direction

b) E along $-y$, B along x, $S \sim -\hat{j} \times \hat{i} \sim \hat{k}$

$\Rightarrow$ z direction

c) E along z, B along $-x$, $S \sim -\hat{k} \times \hat{i} \sim -\hat{j}$

$\Rightarrow$ $-y$ direction

d) E along y, B along $-z$, $S \sim -\hat{j} \times \hat{k} \sim -\hat{i}$

$\Rightarrow$ $-x$ direction

32.10

$$E = E_0 \cos(kx + \omega t)$$

$E_0 = 375 \frac{V}{m}$, $k = 1.99 \times 10^7 \frac{rad}{m}$

$\omega = 5.97 \times 10^{15} \frac{rad}{s}$

a) $V_w = \frac{\omega}{k} = \frac{5.97 \times 10^{15}\, m}{s\ 1.99 \times 10^7} = 3 \times 10^8 \frac{m}{s} = c$

b) $E_0 = 375 \frac{V}{m}$, $B_0 = \frac{E_0}{c} = \frac{375\, V\, s}{m\ 3 \times 10^8\, m}$

$= 1.25 \times 10^{-6}\, T$

c) $\omega = 2\pi f$, $f = \frac{\omega}{2\pi} = \frac{5.97 \times 10^{15}}{s (2\pi)}$

$= 9.5 \times 10^{14} /s$

$\tau = \frac{1}{f} = 1.05 \times 10^{-15} s$

$\lambda = \frac{2\pi}{k} = \frac{2\pi\ m}{1.99 \times 10^7} = 3.14 \times 10^{-7} m$

$= 314\ nm$

not visible

32.18 $\langle S\rangle$ = Poynting flux $= 1.4\times10^{3}\ \frac{W}{m^2}$

$\Rightarrow$ this is time averaged.

$\Rightarrow$ peak is twice the value

$$\Rightarrow 2.8\times10^{3}\ \frac{W}{m^2} = S$$

$$S = \epsilon_0 E^2 c$$

$$E^2 = \frac{S}{\epsilon_0 c} = \frac{2.8\times10^{3}}{8.85\times10^{-12}\ C^2}\ \frac{\overset{N\text{-}m/s}{\cancel{W}}}{m^2}\ \frac{N m^2\ s}{3\times10^{8}\ m}$$

$$= \frac{2.8}{.885(3)}\times10^{6}\ \frac{N^2}{C^2}$$

$$= 1.05\times10^{6}\ \frac{N^2}{C^2}$$

$$E = 1.03\times10^{3}\ \frac{N}{C}$$

$$B = \frac{E}{c} = 1.03\times10^{3}\ \frac{N}{C}\ \frac{1}{3\times10^{8}}\ \frac{s}{m}$$

$$= 3.4\times10^{-6}\ T$$

b) Total power $= 4\pi R^2\langle S\rangle$

$$= 1.4\times10^{3}\ \frac{W}{m^2}\ 4\pi\ 2.25\times10^{22}\ m^2$$

$$= \cancel{4}\ 1.4(.4)\pi\ 2.25\times10^{26}\ W$$

$$= 4.0\times10^{26}\ W$$

32.44 Solar sail.

a) reflective $\Rightarrow$ produces twice the momentum change

b) $\cancel{P_S}$ $3.9 \times 10^{26}\ W$ = power = P_{sun}

$$S = \frac{P_{sun}}{4\pi R^2}$$

pressure of sun = $\frac{S}{c} = \frac{P_{sun}}{4\pi R^2 c}$

$$F_g = \frac{G M_s m}{R^2}, \quad m = 10^4\ Kg, \quad M_s = \text{solar mass}$$

Let A_s = area of sail.

$$\frac{P_{sun} A_s}{4\pi R^2 c} = \frac{G M_s m}{R^2}$$

$$A_s = \frac{G M_s m\, 4\pi\, c}{P_{sun}}$$

$$= \frac{6.67 \times 10^{-11} \frac{Nm^2}{Kg^2}\ 2 \times 10^{30} Kg\ 10^4 Kg\ 4\pi\ 3 \times 10^8 \frac{m}{s}}{3.9 \times 10^{26} \frac{Nm}{s}}$$

$$= \frac{6.67(.8)\pi \times 10^6\ m^2}{1.3}$$

$$= 12.9 \times 10^6\ m^2 = 12.9\ Km^2$$

Both F_g and S fall off as $\frac{1}{r^2}$