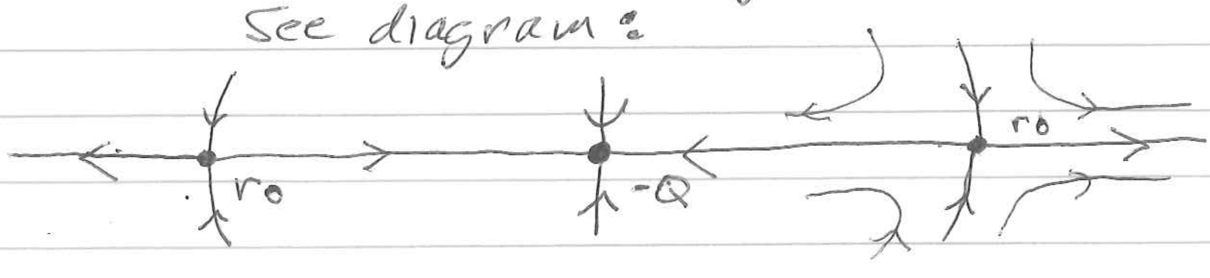
Homework 4 Solutions

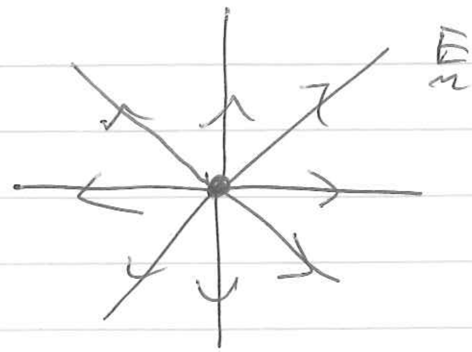
Q 21.7

a) Three point charges have equal magnitude.
The top and bottom charges are positive since $\underline{E}$ points outward from those charges.
The middle charge is negative since $\underline{E}$ points inward

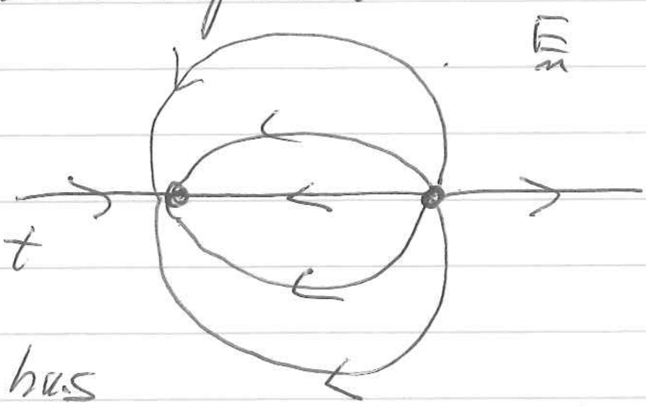
b) Where is $\underline{E}$ smallest? At some radius r_0 ~~along~~ along the symmetry line extending horizontally through the negative charge $\underline{E}$ is zero.
To see this, note that the vertical field E_y is zero along this line since the E_y from the positive charges cancel. At some r_0 the radial field E_r also cancels since at large r, $E_r > 0$ because the positive charges dominate while at small r, $E_r < 0$ since are close to the negative charge.
See diagram:



Q 21.9



For Fig. 21.28 a
E is radially outward.
A positive (negative) charge placed any where will move radially outward (inward) following E everywhere.



For Fig. 21.28b have a dipole.
A positive charge will initially move along E but as the charge moves it ~~q~~ will move to a location where E has a different direction. Thus v and E will not be aligned.

21.6

$$F = \frac{q^2}{4\pi\epsilon_0 r^2} = 3.33 \times 10^{-21} N$$

$$q^2 = F \, 4\pi\epsilon_0 r^2$$

$$= \frac{3.33 \times 10^{-21} \cancel{N} \, C^2 \, (.2\cancel{m})^2}{9 \times 10^9 \cancel{N m^2}}$$

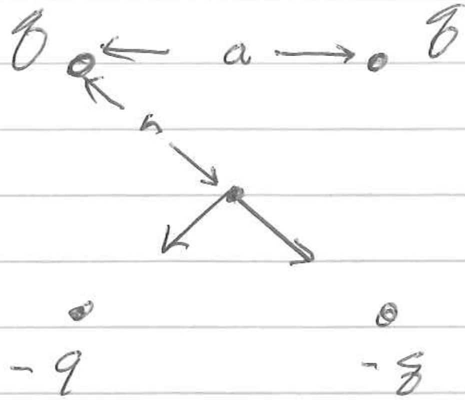
$$q^2 = \frac{3.33(4)}{9} \times 10^{-32} C^2$$

$$q = 1.22 \times 10^{-16} C$$

$$n_e = \# \text{ of electrons} = \frac{1.22 \times 10^{-16}\,C}{1.6 \times 10^{-19}\,C}$$
$$= 760$$

21.38

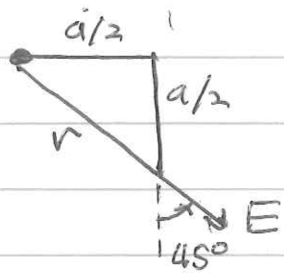
What is the direction of $\underline{E}$ at the center of the square? Magnitude?



At the center the horizontal $\underline{E}$ from the two positive q's and the two negative q's cancels. $\underline{E}$ is downward since $\underline{E}$ from the positive and negative charges is down.

The magnitude of $\underline{E}$ from each charge is

$$E = \frac{q}{4\pi\epsilon_0 r^2} \quad \text{with } r^2 = \frac{a^2}{2}$$

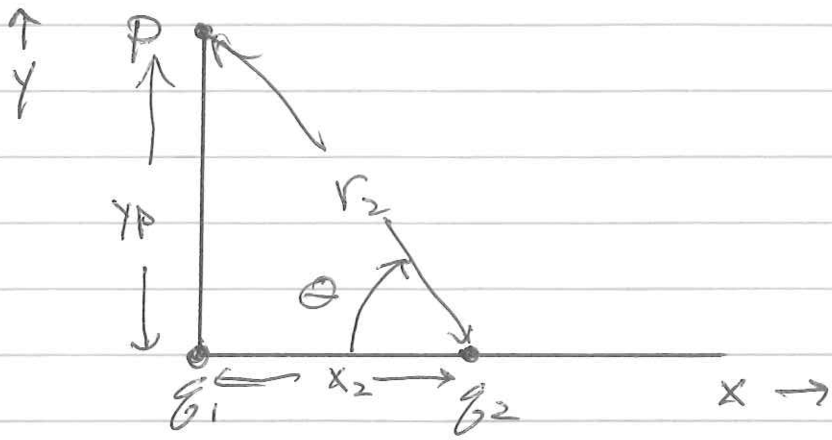


$$r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{2}$$

$$E_y = -E\cos 45^\circ = -\frac{E}{\sqrt{2}} = -\frac{q}{4\pi\epsilon_0\left(\frac{a^2}{2}\right)}\frac{1}{\sqrt{2}}$$
$$= -\frac{q\sqrt{2}}{4\pi\epsilon_0 a^2}$$

$$E_{y\,tot} = 4E_y = -\frac{\sqrt{2}\,q}{\pi\epsilon_0 a^2}$$

21.40



$y_P = 4\,cm$

$x_2 = 3\,cm$

$q_1 = -5\,nC$

$q_2 = 3\,nC$

$$E_1 = \frac{q_1}{4\pi\epsilon_0 y_P^2} = \cancel{5\times10^{-9}\,C}\;\cancel{\frac{9\times10^{9}\,Nm^2}{C^2\;16\times10^{-4}\,m^2}}$$

$$= \frac{45}{16}\times10^4\,\frac{N}{C} = 2.8\times10^4\,\frac{N}{C}$$

$$\underset{\sim}{E}_1 = -2.8\times10^{4}\,\frac{N}{C}\,\hat{j}$$

$$E_2 = \frac{q_2}{4\pi\epsilon_0 r_2^2}\;, \quad r_2^2 = 16\,cm^2 + 9\,cm^2$$

$$= 25\,cm^2$$

$$r_2 = 5\,cm$$

$$E_2 = \cancel{3\times10^{-9}\,C}\;\cancel{9\times10^{9}}\,\frac{Nm^2}{C^2}\,\frac{1}{25\times10^{-4}\,m^2} = \frac{27}{25}\times10^4\,\frac{N}{C}$$

$$E_2 = 1.08\times10^4\,\frac{N}{C}\;, \quad E_{2x} = -E_2\cos\theta$$

$$E_{2x} = -1.08\times10^4\,\frac{N}{C}\,\frac{3}{5} = -6.5\times10^3\,\frac{N}{C}$$

$$E_{2y} = E_2\sin\theta = 1.08\times10^4\,\frac{N}{C}\,\frac{4}{5}$$

$$= 8.6\times10^3\,\frac{N}{C}$$

$$\underset{\sim}{E}_2 = (-6.5\,\hat{i} + 8.6\,\hat{j})\times10^3\,\frac{N}{C}$$

$$E_{tot} = \underset{\sim}{E}_1 + \underset{\sim}{E}_2 = -6.5\times10^3\,\frac{N}{C}\,\hat{i} - 1.9\times10^4\,\frac{N}{C}\,\hat{j}$$

21.60

Force balance in vertical direction

$$mg = T\cos\theta \Rightarrow T = \frac{mg}{\cos\theta}$$

Horizontal force balance

$$T\sin\theta = \frac{q^2}{4\pi\epsilon_0 (2L\sin\theta)^2} = \frac{mg}{\cos\theta}\sin\theta$$

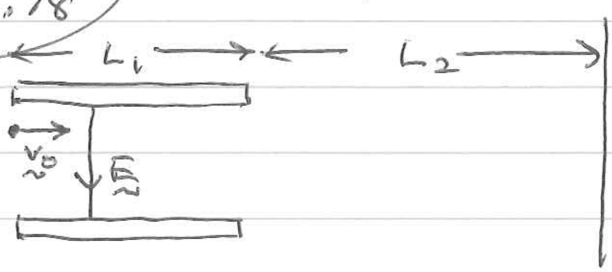
$d = 2L\sin\theta$

$$= \frac{mg\,d}{2L\cos\theta}$$

$$\frac{q^2}{4\pi\epsilon_0}\frac{1}{d^2} = \frac{mg\,d}{2L} \quad \text{where } \cos\theta \simeq 1$$

$$d = \left(\frac{q^2 L}{2\pi\epsilon_0 mg}\right)^{1/3}$$

21.78



$L_1 = 26\text{ cm}$
$L_2 = 56\text{ cm}$
$v_0 = 5\times10^3\text{ m/s}$

$$t_1 = \text{time in } E = \frac{.26\text{ m}}{5\times10^3\text{ m/s}} = 5.2\times10^{-5}\text{ s}$$

t_2 = time outside of E

$$= \frac{.56\text{ m}}{5\times10^3\text{ m/s}} = 1.12\times10^{-4}\text{ s}$$

V_1 = downward velocity after acceleration

$$= \frac{qE}{m}t_1$$

d_1 = downward distance in E region

$$= \frac{1}{2}\frac{qE}{m}t_1^2$$

d_2 = downward distance ~~in~~ outside of E

$$= v_1 t_2$$

$$d_1 + d_2 = d = v_1 t_2 + \frac{1}{2}\frac{qE}{m} t_1^2$$

$$d = \frac{qE}{m} t_1 t_2 + \frac{1}{2}\frac{qE}{m} t_1^2$$

$$= \frac{q}{m}\left[t_2 + \frac{1}{2}t_1\right] E t_1$$

$$1.25 \times 10^{-2}\,\text{m} = \left(\frac{q}{m}\right)\left(1.12 \times 10^{-4}\,\text{s} + \frac{1}{2}\,5.2 \times 10^{-5}\,\text{s}\right)$$

$$\times\ 800 \frac{\text{kg m}}{\text{s}^2\,\text{C}}\ 5.2 \times 10^{-5}\,\text{s}$$

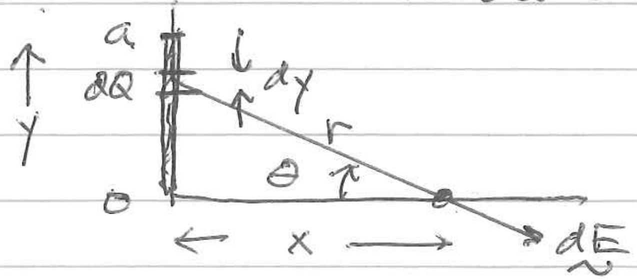
$$1.25 = \left(\frac{q}{m}\right) 1.38 \times 10^{-4}\ .8\ \ 5.2\ \frac{\text{kg}}{\text{C}}$$

$$\frac{q}{m} = \frac{1.25}{1.38(.8)(.52)} \times 10^{3}\ \frac{\text{C}}{\text{kg}}$$

$$= 2.18 \times 10^{3}\ \frac{\text{C}}{\text{kg}}$$

21.80 a)

$$dQ = \frac{Q}{a}\,dy \quad , \quad dE = \frac{dQ}{4\pi\epsilon_0 r^2}$$



$d\underset{\sim}{E}$ = electric field from small charge dQ

$$dE_x = dE\cos\theta \qquad \cos\theta = \frac{x}{r}$$

$$dE_y = -dE\sin\theta \qquad \sin\theta = \frac{y}{r}$$

$$r = (x^2 + y^2)^{1/2}$$

$$E_x = \int dE_x = \int_0^a dy \, \frac{Q}{a} \, \frac{1}{4\pi\epsilon_0 (x^2+y^2)^{3/2}} \, x$$

$$\int dy \, \frac{1}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

$$E_x = \frac{Q}{4\pi\epsilon_0 \cancel{a}} \, \frac{\cancel{a}}{x^2 (x^2+a^2)^{1/2}} \, x$$

$$E_x = \frac{Q}{4\pi\epsilon_0 \, x (x^2+a^2)^{1/2}}$$

$$E_y = \int dE_y = -\frac{Q}{a} \int_0^a dy \, \frac{1}{4\pi\epsilon_0 (x^2+y^2)^{3/2}} \, y$$

$$\int dy \, \frac{y}{(x^2+y^2)^{3/2}} = -\frac{1}{(x^2+y^2)^{1/2}}$$

$$E_y = + \frac{Q}{4\pi\epsilon_0 a} \left[\frac{1}{(x^2+a^2)^{1/2}} - \frac{1}{x} \right]$$

b)

$$F_x = -q E_x \, , \quad F_y = -q E_y$$

c) For $x \gg a$, $F_x = -q E_x \simeq -q \dfrac{Q}{4\pi\epsilon_0 x^2}$

$$F_y \simeq + \frac{qQ}{4\pi\epsilon_0 a} \left[1 - \frac{1}{\left(1 + \frac{a^2}{x^2}\right)^{1/2}} \right] \frac{1}{x}$$

Use Taylor series expansion

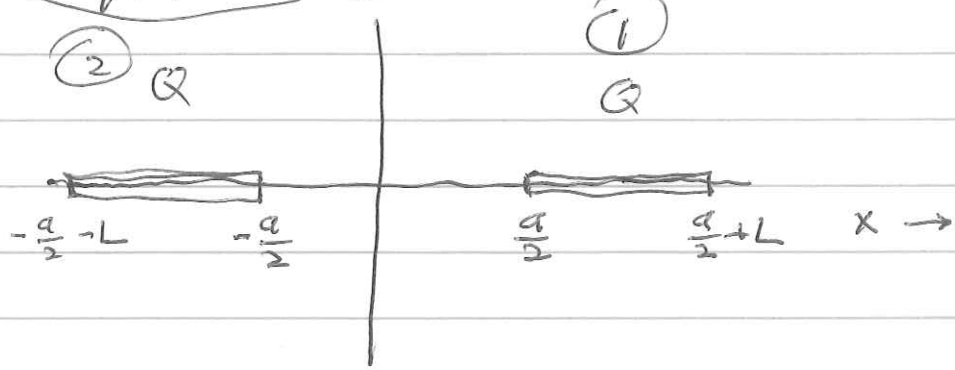
$$\frac{1}{(1+\epsilon)^{1/2}} \simeq 1 - \frac{1}{2}\epsilon \quad \text{for } \epsilon \ll 1$$

$$F_y \simeq \frac{qQ}{4\pi\epsilon_0 a}\,\frac{1}{x}\left[\cancel{x} - \left(\cancel{x} - \frac{1}{2}\frac{a^2}{x^2}\right)\right]$$

$$= \frac{qQ}{8\pi\epsilon_0}\,\frac{a}{x^3}$$

<u>Extra</u>

29.97 a)



Calculate $\underset{\sim}{E}$ from rod 2 along the positive x axis.

Consider a small charge segment dQ at a location x'. Calculate dE from this dQ

$$dE = \frac{dQ}{4\pi\epsilon_0 (x-x')^2}\,, \qquad dQ = \frac{Q}{L}\,dx'$$

$$E = \int dE = \frac{Q}{L\,4\pi\epsilon_0}\int_{-\frac{a}{2}-L}^{-a/2} dx'\,\frac{1}{(x-x')^2}$$

$$= \frac{Q}{4\pi\epsilon_0 L}\left(\frac{1}{x-x'}\right)\Bigg|_{-\frac{a}{2}-L}^{-a/2}$$

$$= \frac{Q}{4\pi\epsilon_0 L}\left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]$$

b) Calculate the force on 1 due to 2.
Consider a small segment $dQ = \frac{Q}{L}dx$ of rod 1.

$$dF = dQ\, E$$

$$F = \frac{Q}{L}\int_{\frac{a}{2}}^{\frac{a}{2}+L} dx \, \frac{Q}{4\pi\epsilon_0 L}\left[\frac{1}{x+\frac{a}{2}} - \frac{1}{x+\frac{a}{2}+L}\right]$$

$$= \frac{Q^2}{4\pi\epsilon_0 L^2}\left[\ln\left(\frac{a+L}{a}\right) - \ln\left(\frac{a+2L}{a+L}\right)\right]$$

$$F = \frac{Q^2}{4\pi\epsilon_0 L^2}\ln\left[\frac{(a+L)^2}{a(a+2L)}\right]$$

c) $a \gg L$

$$\ln\frac{(a+L)^2}{a(a+2L)} = \ln\left[\frac{\left(1+\frac{L}{a}\right)^2}{1+\frac{2L}{a}}\right]$$

$$= 2\ln\left(1+\frac{L}{a}\right) - \ln\left(1+\frac{2L}{a}\right)$$

$$= 2\left(\cancel{\frac{L}{a}} - \frac{1}{2}\frac{L^2}{a^2}\right) - \left(\cancel{\frac{2L}{a}} - \frac{1}{2}\frac{4L^2}{a^2}\right)$$

$$= -\frac{L^2}{a^2} + \frac{2L^2}{a^2} = \frac{L^2}{a^2}$$

$$F \simeq \frac{Q^2}{4\pi\epsilon_0 a^2}$$