Homework 2 Solutions

Q 22.4 A region of space is bounded by an imaginary closed surface with no enclosed charge. Is $\underset{\sim}{E}$ always zero everywhere on the surface?
No. Charge outside the closed surface will produce a non-zero field. $\underset{\sim}{E}$ will be zero only if there is no charge anywhere or if the surface is in a conductor or in a void in a conductor.

22.2 The electric field flux through a surface A is given by

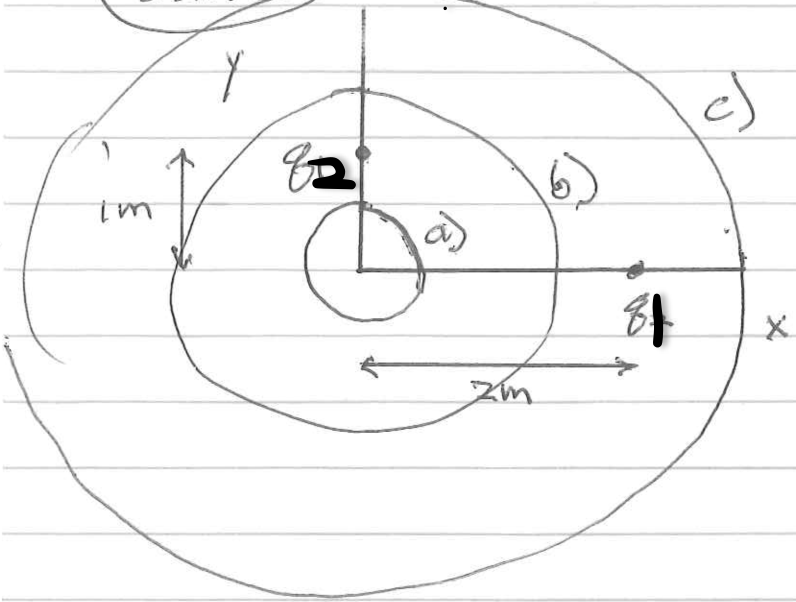
$$\Phi_E = \int_A \underset{\sim}{E} \cdot d\underset{\sim}{A}$$ with $d\underset{\sim}{A}$ normal to the surface

$$\int_A \underset{\sim}{E} \cdot d\underset{\sim}{A} = E A \cos\theta$$ with $\theta = 70°$ the angle between $\underset{\sim}{E}$ and $\underset{\sim}{A}$.

$$\Phi_E = 0.24\,m^2 \; 90\,\frac{N}{C} \; 0.342$$

$$= 7.4\,\frac{Nm^2}{C}$$

22.10



a) $r = 0.5\,m$ encloses no charge so $\Phi_E = 0$

b) $r = 1.5\,m$ encloses $q_2 = -6\,nC$

$$\Phi_E = -\frac{6 \times 10^{-9}\,C \;\; Nm^2}{8.85 \times 10^{-12}\,C^2}$$

$$= \overset{-6.78}{\cancel{4.5}} \times 10^{2}\ \frac{Nm^2}{C}$$

c) $r = 2.5\,m$ encloses both charges

$$\Phi_E = -\frac{2 \times 10^{-9}\,C \;\; Nm^2}{8.85 \times 10^{-12}\,C^2} = -2.26 \times 10^{2}\ \frac{Nm^2}{C}$$

22.26 Have a square insulating sheet 80cm on a side with 4.5 nC of charge.

a) At 0.1 mm above sheet, the sheet acts ~~seems~~ like an infinite sheet.

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{see eq. } 21.12)$$

$$= \frac{4.5 \times 10^{-9}\,C \quad Nm^2}{(2).64\,m^2 \;\; 8.85 \times 10^{-12}\,C^2}$$

$$= \overset{3.97}{\cancel{7.9}} \times 10^{2}\ N/C$$

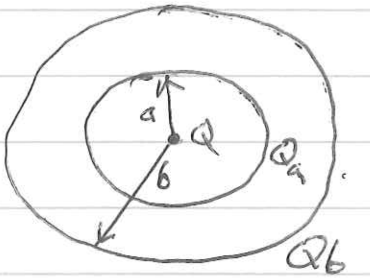
b) 100 m above the sheet, the sheet acts like a point charge.

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{4.5 \times 10^{-9}\,C \;\; 9 \times 10^{9}\,Nm^2}{10^4\,m^2 \quad C^2}$$

$$= 4.05 \times 10^{-3}\ N/C$$

c) What if the sheet were a conductor? Near the center of the sheet for part a) the value of E would drop significantly since most of the charge would move to the edges of the sheet. The value of E in b) would not change significantly since the sheet is still like a point charge at 100 m.

22.42



For $r < a$,

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

since the ~~cha~~ electric field from the charge on the conductor all cancels (by symmetry).

For $a < r < b$, $E = 0$ since inside the conductor. The electric flux through a Gaussian surface inside the conductor is zero so the net charge inside is zero. This means that the total charge on the inner surface at $r = a$ is $-Q$.

For $r > b$, the total charge enclosed in a Gaussian surface is $Q - 3Q = -2Q$

$$E = \frac{-2Q}{4\pi\epsilon_0 r^2}$$

The total charge on the outer surface is $-2Q$ since $-Q$ is on the inner surface and total charge on the conductor is $-3Q$.

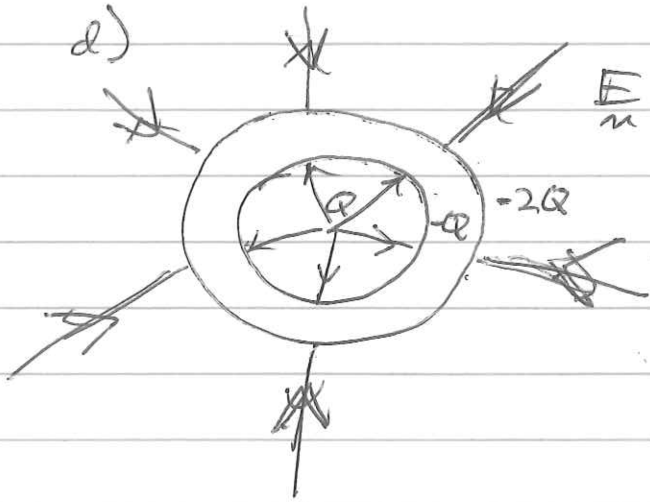
b) On the inner surface

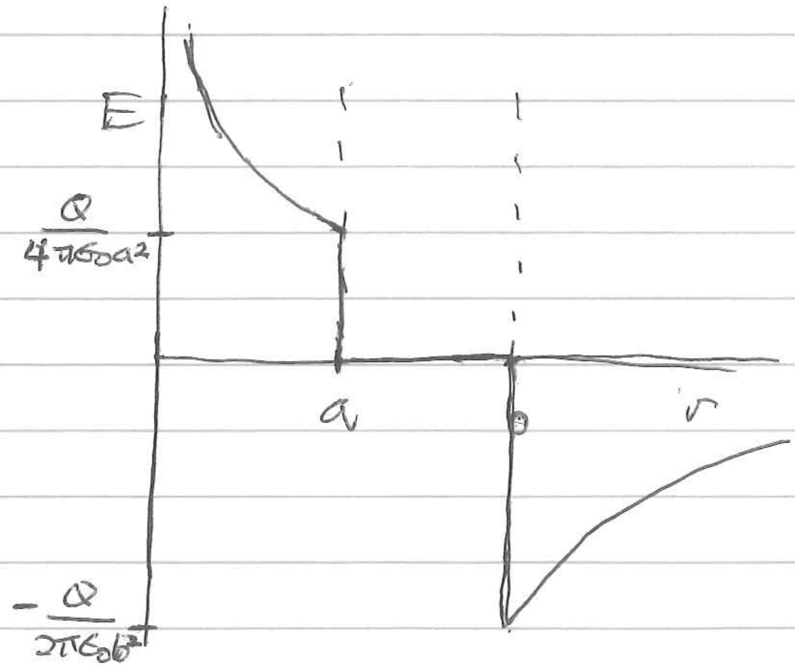
$$\sigma = \frac{-Q}{4\pi a^2}$$

c) On the outer surface

$$\sigma = -\frac{2Q}{4\pi b^2}$$

d)





22.50 Solid insulating sphere with radius R.
Charge density is non-uniform

$$\rho = \rho_0 \left(1 - \frac{r}{R}\right)$$

with ρ_0 a constant.

a) Find $E(r)$ for $r < R$. Use Gauss' Law

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = 4\pi r^2 E = \frac{q(r)}{\epsilon_0}$$

since E is radial and only a function of r.

$$q(r) = \int_0^r 4\pi r'^2 dr' \rho_0 \left(1 - \frac{r'}{R}\right)$$

$$= 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$= \frac{4\pi\rho_0}{3} r^3 \left(1 - \frac{3}{4}\frac{r}{R}\right)$$

$$\cancel{4\pi} r^2 E = \frac{1}{\epsilon_0}\left(\frac{\cancel{4\pi}\rho_0}{3} r^3\right)\left(1 - \frac{3}{4}\frac{r}{R}\right)$$

$$E = \frac{\rho_0}{3\epsilon_0} r \left(1 - \frac{3}{4}\frac{r}{R}\right)$$

b) $r > R$. Total charge $q_{tot} = q(R)$

$$q_{tot} = \frac{\cancel{4}\pi\rho_0}{3} R^3 \frac{1}{\cancel{4}} = \frac{\pi\rho_0 R^3}{3}$$

E acts like a point charge

$$E = \frac{\cancel{\pi}\rho_0 R^3}{(3)\, 4\cancel{\pi}\epsilon_0} \frac{1}{r^2} = \frac{\rho_0 R^3}{12\epsilon_0} \frac{1}{r^2}$$

c) Where does E take on its maximum value? For $r > R$ E is decreasing with r. To find the max value, take the derivative of E for $r < R$ and set to zero,

$$\frac{dE}{dr} = \frac{\rho_0}{3\epsilon_0} \frac{d}{dr}\left[r\left(1 - \frac{3}{4}\frac{r}{R}\right)\right] = 0$$

$$1 - \frac{3}{4}\frac{2r}{R} = 0 \Rightarrow \frac{r}{R} = \frac{2}{3}$$

22.54 Consider a long cylinder of radius R with a uniform charge density except in a cylindrical hole of radius a that is a distance b from the main cylinder.

First calculate E for a solid cylinder for $r < R$. From Gauss' Law with Gaussian cylinder of length l. $\Rightarrow$ E radially outward

$$E\, 2\pi r l = \frac{1}{\epsilon_0} q(r)$$

$$q(r) = l \rho \pi r^2$$

$$E\, 2\cancel{\pi} r \cancel{l} = \cancel{l} \rho \cancel{\pi} r^2 \frac{1}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0}$$

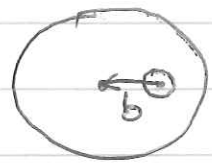
Now calculate $\underset{\sim}{E}$ from a plug of material with a charge density of $-\rho$

$$E_{-\rho} = -\frac{\rho r}{2\epsilon_0}$$

where r is now centered on the axis of the plug.

As suggested in problem 22.53 write $\underset{\sim}{E}$ in the reference ~~frame~~ of the hole. In this frame the electric field from ρ is

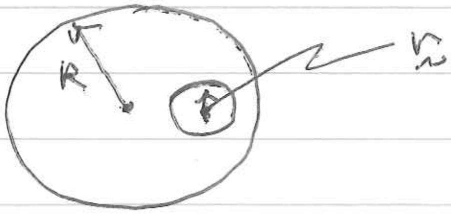
$$\underset{\sim}{E} = \rho \frac{\underset{\sim}{r} - \underset{\sim}{b}}{2\epsilon_0}$$



and

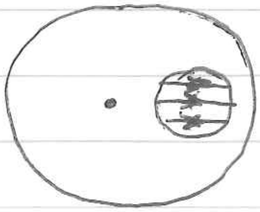
$$\underset{\sim}{E}_{-\rho} = -\rho \frac{\underset{\sim}{r}}{2\epsilon_0}$$

where $\underset{\sim}{r}$ is a radius vector from the center of the hole



$$\underset{\sim}{E}_{tot} = \underset{\sim}{E} + \underset{\sim}{E}_{-\rho}$$

$$= \frac{\rho}{2\epsilon_0}\left(-\underset{\sim}{b}\right)$$



$\underset{\sim}{E}_{tot}$ points ~~toward~~ outward from the center of the cylinder with

$$E_{tot} = \frac{\rho b}{2\epsilon_0}$$