Homework 3 Solutions

Q 23.5

The potential difference between any two points is given by

$$V_B - V_A = -\int_{x_A}^{x_B} \underset{\sim}{E} \cdot dx$$

Since $\underset{\sim}{E} = 0$, $V_B = V_A$. For a different path $\underset{\sim}{E}$ need not be zero at every point along the path as long as the integral remains zero.

23.14

A particle with charge $q = 4.2\ nC$ is released from rest in a uniform $\underset{\sim}{E}$. KE is $2.2 \times 10^{-6}\ J$ after moving $L = 6\ cm$.

a) Work done is the KE.

b) $$KE = q\,\Delta V$$

$$\Longrightarrow \Delta V = KE/q$$

$$= \frac{2.2 \times 10^{-6}\ J}{4.2 \times 10^{-9}\ C} = 5.2 \times 10^{2}\ \frac{J}{C}$$

c)

$$E = \Delta V / L = \frac{5.2 \times 10^{2}\ J/C}{.06\ m}$$

$$= 8.7 \times 10^{3}\ N/C$$

23.4

Push two protons from $R_1 = 2\times10^{-10}$ m to 3×10^{-15} m $= R_2$. The potential from a ~~proton~~ charge at distance R is

$$V = \frac{q}{4\pi\epsilon_0 R}$$

The difference in potential between the two points is

$$\Delta V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

a) The work is $q\Delta V$,

$$W = \text{work} = (1.6\times10^{-19}\,C)^2\, 9\times10^{9}\,\frac{Nm^2}{C^2} \times \left(\frac{1}{3\times10^{-15}\,m} - \frac{1}{2\times10^{-10}\,m}\right)$$

$$\simeq (1.6\times10^{-19})^2\, 3\times10^{24}\ Nm$$

$$= 7.7\times10^{-14}\ Nm$$

b) Released from R_2. What is velocity at R_1?

Note that both particles are accelerated.

$$2\,\tfrac{1}{2} m v^2 = W$$

$$v^2 = \frac{(7.7\times10^{-14}\ Nm)}{1.67\times10^{-27}\ kg} = 4.6\times10^{13}\ \frac{m^2}{s^2}$$

$$v = 6.8\times10^{6}\ \frac{m}{s}$$

23.36

Parallel conducting plates with
$\sigma = 47\,nC/m^2$.

a) What is E between the plates

$$E = \frac{\sigma}{\epsilon_0} = \frac{47\,nC}{m^2}\;\frac{Nm^2}{8.85\times10^{-12}\,C^2}$$

$$= 5.3\times10^3\ N/C$$

b) Potential difference? $d = 2.2cm$

$$\Delta V = Ed$$

$$= 5.3\times10^3\,\frac{N}{C}\;.022m$$

$$= 1.17\times10^2\ J/C$$

c) If plate separation doubles, with σ constant,

E unchanged.
ΔV doubles

23.41

Two spherical hollow conducting shells with radius r_a and r_b with $r_a < r_b$. For $r > r_b$, $\vec{E} = 0$ from Gauss' law since total enclosed charge is zero.

$$V(r_b) = -\int_{\infty}^{r_b} E\,dr = 0$$

and $V(r) = 0$ for all $r > r_b$.

For $r > r_a$ but $r < r_b$, from Gauss' Law

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

with q the total charge enclosed (on inner sphere). For ~~r~~ $r_b > r > r_a$

$$V(r) = -\int_{r_b}^{r} dr \, \frac{q}{4\pi\epsilon_0 r^2}$$

$$= \frac{q}{4\pi\epsilon_0} \frac{1}{r} \Big|_{r_b}^{r} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r_b}\right)$$

$\Rightarrow$ for $r = r_a$

b)

$$V(r_a) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

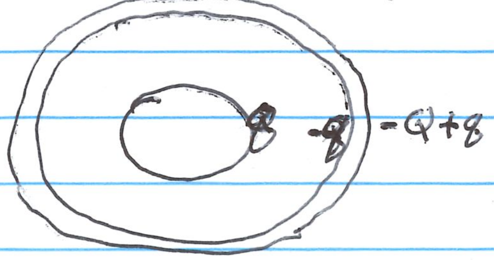
For $r < r_a$, $E = 0$ from Gauss' law since no charge enclosed in Gaussian surface. Thus, $V = \text{const}$ for $r < r_a$

$$V(r) = V(r_a) \text{ for } r < r_a$$

Can write $E(r)$ for $r_b > r > r_a$,

$$E = \frac{V(r_a)}{\frac{1}{r_a} - \frac{1}{r_b}} \frac{1}{r^2}$$

e) Charge on outer sphere is $-Q$



The outer shell will have charge $-q$ on the inside and $-Q+q$ on the outside.

E in the region $r_b > r > r_a$ remains unchanged because ~~Gauss' that res~~ the charge enclosed in a Gaussian surface ~~for~~ in this region remains q. Thus the potential difference between the two plates, which is the integral of E between r_b and r_a, is the same. For $r > r_b$, using Gauss' law

$$E = \frac{1}{4\pi\epsilon_0 r^2}(q - Q) \neq 0$$

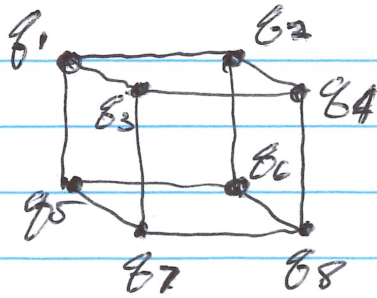
23.57 The potential energy U is the sum of the energies of all possible pairs

$$U = \sum_{i \neq j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

with r_{ij} = separation of charges q_i and q_j.

r_{ij} takes on three possible values, d, $\sqrt{2}\,d$ (diagonal across faces), $\sqrt{3}\,d$ (diagonal across entire crystal).

To help with counting label all of the charges $q_1, q_2 \cdots q_8$.



$$4\pi\epsilon_0 U = q_1 \sum_{i>1} \frac{q_i}{r_{1i}} + q_2 \sum_{i>2} \frac{q_i}{r_{2i}} + q_3 \sum_{i>3} \frac{q_i}{r_{3i}} + q_4 \sum_{i>4} \frac{q_i}{r_{4i}} + q_5 \sum_{i>5} \frac{q_i}{r_{5i}} + q_6 \sum_{i>6} \frac{q_i}{r_{6i}} + \frac{q_7 q_8}{r_{78}}$$

$$\sum_{i>1} \frac{q_i}{r_{1i}} = q\left(\frac{1}{d} + \frac{1}{d} - \frac{1}{\sqrt{2}d} + \frac{1}{d} - \frac{1}{\sqrt{2}d} - \frac{1}{\sqrt{2}d} + \frac{1}{\sqrt{3}d}\right)$$

$$= \frac{q}{d}\left(3 - \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)$$

$$\sum_{i>2} \frac{q_i}{r_{2i}} = \frac{q}{d}\left(\frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} - 1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{q}{d}\left(-2 + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$$

$$\sum_{i>3} \frac{q_i}{r_{3i}} = \frac{q}{d}\left(-1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} - 1 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{q}{d}\left(-2 + \frac{2}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$$

$$\sum_{i>4} \frac{q_i}{r_{4i}} = \frac{q}{d}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1\right)$$

$$= \frac{q}{d}\left(1 - \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)$$

$$\sum_{i>5} \frac{q_i}{r_{5i}} = \frac{q}{d}\left(-1 - 1 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{q}{d}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

$$\sum_{i>6} \frac{q_i}{r_{6i}} = \frac{q}{d}\left(-\frac{1}{\sqrt{2}} + 1\right)$$

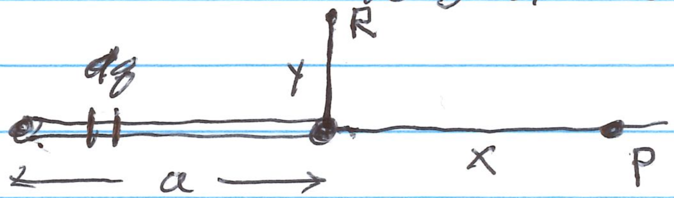
$$\sum_{i>7} \frac{q_i}{r_{7i}} = \frac{q}{d}\left(+1\right)$$

$$\frac{4\pi\epsilon_0 U d}{q} = -\left(3 - \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)$$
$$+ \left(-2 + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$$
$$+ \left(-2 + \frac{2}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$$
$$- \left(1 - \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)$$
$$+ \left(-2 + \frac{1}{\sqrt{2}}\right)$$
$$- \left(1 - \frac{1}{\sqrt{2}}\right)$$
$$-(1) \qquad U = \frac{q}{\pi\epsilon_0 d}(-1.46)$$

$$= -12 + \frac{12}{\sqrt{2}} - \frac{4}{\sqrt{3}} < 0$$

Since $U < 0$, energy is required to break it apart and so are stable.

23-71 Charge Q is distributed along a rod of length "a".



a) Calculate V at P. Consider a small charge $dq = \frac{Q}{a}dx'$. The potential from dq at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{X - X'}$$

where dq is at x'.

$$V = \int_{-a}^{0} \frac{1}{4\pi\epsilon_0} \frac{Q}{a} dx' \frac{1}{x-x'}$$

$$= -\frac{Q}{4\pi\epsilon_0 a} \ln(x-x') \Big|_{-a}^{0}$$

$$= \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x+a}{x}\right)$$

b) Calculate V at R

$$V = \int_{-a}^{0} \frac{1}{4\pi\epsilon_0} \frac{Q}{a} dx' \frac{1}{(y^2+x'^2)^{1/2}}$$

$$\int_{-a}^{0} dx' \frac{1}{(y^2+x'^2)^{\frac{1}{2}}} = \ln\left(x' + (y^2+x'^2)^{\frac{1}{2}}\right) \Big|_{-a}^{0}$$

$$= \ln \frac{y}{(y^2+a^2)^{1/2} - a}$$

$$V = \frac{Q}{4\pi\epsilon_0 a} \ln \frac{y}{(y^2+a^2)^{1/2} - a}$$

for $y >> a$

$$V = \frac{Q}{4\pi\epsilon_0 a} \ln \frac{1}{\cancel{1+\frac{1}{2}\frac{a^2}{y^2}} - \frac{a}{y}}$$

$$\simeq \frac{Q}{4\pi\epsilon_0 a} \frac{a}{y} = \frac{Q}{4\pi\epsilon_0 y}$$