Homework 4 Solutions

Q 24.6 A parallel plate capacitor is connected to a battery. The plate separation is doubled. How does E change?

$\Rightarrow$ Since the total potential drop remains the same and $E = V/d$ with d the separation, E must be reduced by a factor of 2.

$\Rightarrow$ What about the energy?

$U = \left(\frac{1}{2}\epsilon_0 E^2\right) Ad$

with A the plate area and d the separation. Since E is reduced by 2 and d increases by 2, the total energy drops by 2.

24.2 Parallel plate capacitor with $A = 9.82\ cm^2$ and $d = 3.28\ mm$.

a) Capacitance?

$$C = \frac{A\epsilon_0}{d} = \frac{(9.82 \times 10^{-4})\, m^2 \; 8.85 \times 10^{-12} \frac{C^2}{Nm^2}}{3.28 \times 10^{-3}\ m}$$

$$= \frac{(.982)\, 8.85}{3.28} \times 10^{-12}\ F$$

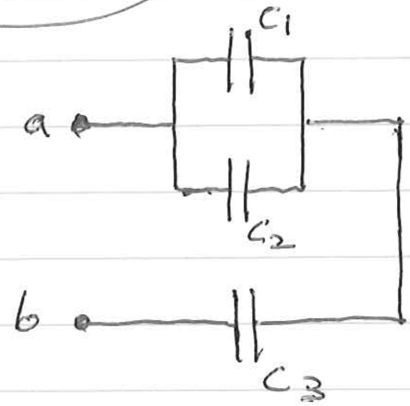
$$= 2.6\ pF$$

b) Voltage? $V = \frac{Q}{C} = \frac{4.35 \times 10^{-8}\ C}{2.6 \times 10^{-12}\ F} = 1.67 \times 10^{4}\ V$

c) Electric field?

$$E = \frac{V}{d} = \frac{1.67 \times 10^{4}\,V}{3.28 \times 10^{-3}\,m} = 5.1 \times 10^{6}\,\frac{V}{m}$$

24.20 Consider the circuit shown



$C_1 = 6\mu F$
$C_2 = 3\mu F$
$C_3 = 5\mu F$

a) Charges on C_1?
Voltage drop across C_1 and C_2 are equal

$$\Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\Rightarrow Q_1 = \frac{C_1}{C_2} Q_2 = \frac{6}{3}\, 30\mu C$$

$$= 60\mu C$$

Charge on C_3?

$$Q_3 = Q_1 + Q_2 = 60\mu C + 30\mu C = 90\mu C$$

b) Applied voltage V_{ab}? First find total capacitance

$C_{12} = C_1 + C_2$ = capacitance of C_1, C_2 together.
C_{12} and C_3 are in series. Total capacitance C is

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_{12}} = \frac{1}{5\mu F} + \frac{1}{9\mu F}$$

$$C = \frac{45}{14}\mu F$$

$$V = \frac{Q}{C} = \frac{Q_3}{C} = \frac{90\mu C}{\frac{45}{14}\mu F} = 28\,V$$

Can also find $V = V_2 + \frac{Q_3}{C_3} = \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = 28\,V$

24.60 Consider capacitor with area A and separation d. Insert metal plate with thickness "a". C_0 is the original capacitance.

$$C_0 = \frac{\epsilon_0 A}{d}$$

a) What is the capacitance C?

$E = \frac{Q}{A\epsilon_0}$ is unchanged in gap regions

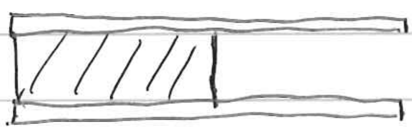
The voltage drop is $V = E(d-a) = \frac{Q(d-a)}{A\epsilon_0} = \frac{Q}{C}$

$$C = \frac{A\epsilon_0}{d-a}$$

b) ϵ. $\frac{C}{C_0} = \frac{d}{d-a}$

c) As $a \to 0$, $C \to C_0$. As $a \to d$, $C \to \infty$.

24.64



Half of capacitor filled with plexiglass with $\epsilon = 3.4\,\epsilon_0$

a) What is the capacitance? The voltage across the air and plexiglass regions are equal since the plates have constant voltage.

$\Rightarrow$ therefore the two regions are like capacitors in parallel

$C = C_p + C_a$ with C_p the capacitance of the plexiglass region and C_a that of air.

Each have half of the area of the original given by $A = 144\ cm^2$. Thus.

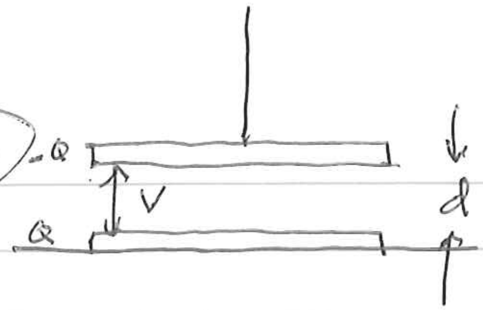
$$C_p = \frac{\epsilon A}{2d} \quad \text{and} \quad C_a = \frac{\epsilon_0 A}{2d}$$

Thus,
$$C = \frac{\epsilon_0 A}{2d}(1 + 3.4)$$
$$= 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \frac{144 \times 10^{-4}\ m^2}{9 \times 10^{-3}\ m}(4.4)$$
$$= \frac{8.85}{9} 1.44(4.4) \times 10^{-11}\ F$$
$$= 62.3\ pF$$

b) Energy stored?
$$U = \frac{1}{2} C V^2 = \frac{1}{2} 62.3 \times 10^{-12} F (18\ V)^2$$
$$= \frac{62.3(.18)^2}{2} \times 10^{-8}\ J$$
$$= 1.01 \times 10^{-8}\ J$$

c) No plexiglass? $U = ?$
C is reduced to $\epsilon_0 A/d$ compared with $2.2\ \epsilon_0 A/d$ with plexiglass. With V the same, the energy is reduced by 2.2 to
$$U = 4.6 \times 10^{-9}\ J$$

24-65



Voltage fixed at V

a) Tension in the cable?

The electric field at the top plate due to only the charge on the bottom plate is

$$E_Q = \frac{\sigma}{2\epsilon_0} = \frac{1}{2}\frac{V}{z}$$

Force on the upper plate is downward

$$F = QE_Q = \frac{1}{2}\frac{V}{z}Q$$

As z varies, Q will change to maintain a constant V.

$$Ez = V = \frac{Q}{A\epsilon_0}z$$

so $$F = \frac{1}{2}\frac{V}{z}\frac{A\epsilon_0}{z}V = \frac{1}{2}\frac{\pi r^2\epsilon_0 V^2}{z^2}$$

a) Initial tension balances the attractive force

$$T = \frac{\pi r^2\epsilon_0}{2d^2}V^2$$

b) Raise top plate to $z = 2d$.

$$W = \int_d^{2d} F(z)\,dz = \frac{\pi r^2\epsilon_0 V^2}{2}\int_d^{2d} dz\,\frac{1}{z^2}$$

$$= \frac{\pi r^2\epsilon_0 V^2}{2}\left(\frac{1}{d} - \frac{1}{2d}\right)$$

$$= \frac{\pi r^2\epsilon_0 V^2}{4d}$$

c) Energy stored when $z = d$

$$U_i = \frac{1}{2} C V^2 = \frac{1}{2} \frac{\epsilon_0 \pi r^2}{d} V^2$$

d) Energy stored when $z = 2d$

$$U_f = \frac{1}{4} \frac{\epsilon_0 \pi r^2}{d} V^2$$

e) The stored energy is reduced by a factor of 2. To maintain the voltage charge had to flow ~~from~~ to the battery since

$$Q = \frac{V A \epsilon_0}{z} \Rightarrow \Delta Q = -\frac{V A \epsilon_0}{2d}$$

$$W + U_i - U_f = \frac{\pi r^2 \epsilon_0 V^2}{2d} = \Delta Q V$$

$\Rightarrow$ energy given back to battery