Homework 5 Solutions

Q 25.10 a) A wire has the same diameter on two sides of a resistor. ~~What is the~~ The electron drift speed on the two sides is the same because the total current is the same. There is a potential drop across the resistor but that does not impact the drift speed outside of the resistor.

b) The potential energy of the electron entering the resistor is higher than that exiting. An electric field is required to drive the current in the resistor. The electric field points down the potential.

25.4 18 guage copper wire has a diameter of 1.02 mm. Current density $J = 3.2 \times 10^6 \, A/m^2$.

a) The current is

$$I = JA = 3.2 \times 10^6 \frac{A}{m^2} \frac{\pi}{4} \left(1.02 \times 10^{-3}\right)^2 m^2$$

$$= \frac{3.2(\pi)}{4} (1.02)^2 A = 2.6 A$$

b) drift velocity

$$J = n e v_d \Rightarrow v_d = \frac{J}{ne}$$

$$v_d = \frac{3.2 \times 10^6 \frac{C}{s} \frac{1}{m^2}}{8.5 \times 10^{28} / m^3 \; 1.6 \times 10^{-19} C}$$

$$= \frac{2.0}{.85} \times 10^{-4} \frac{m}{s} = 2.35 \times 10^{-4} \, m/s$$

(25.12) a) what E is required to produce a 4.5 A current in a Cu wire with a diameter of 2.05 mm

$$\Rightarrow J = \frac{4.5\,A\,(4)}{\pi\,(2.05\times10^{-3})^2\,m^2}$$

$$= \frac{18}{\pi\,(2.05)^2}\times10^{6}\ \frac{A}{m^2}$$

$$= 1.36\times10^{6}\ \frac{A}{m^2}$$

In a conductor

$$E = \rho J = 1.72\times10^{-8}\,\Omega m\ 1.36\times10^{6}\ \frac{A}{m^2}$$

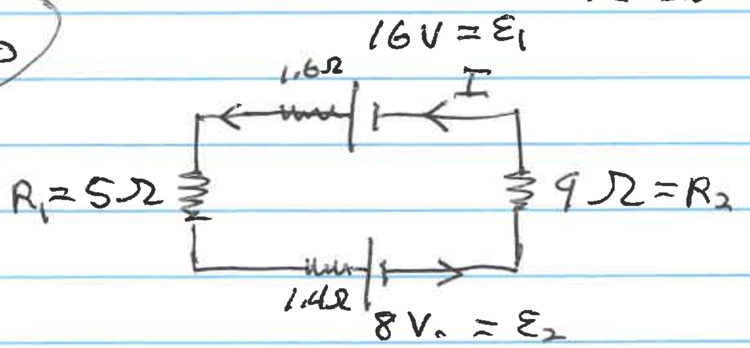
$$= 2.35\times10^{-2}\ \frac{N}{C}$$

b) Silver?

$$E = 2.35\times10^{-2}\ \frac{N}{C}\ \frac{1.47}{1.72}$$

$$= 2.0\times10^{-2}\ N/C$$

(25.30)



I counter clockwise since top $\mathcal{E}$ stronger

loop rule clockwise

$$(16\,V - 8\,V) - I(1.6 + 5 + 1.4 + 9)\,\Omega$$

$$I = \frac{8}{17}\,A$$

b) terminal voltage of top battery

$$V_{ab} = \left[16 - \tfrac{8}{17}(1.6)\right]V = 15.2\,V$$

$$V_c = -\frac{8}{17}\,A\,(5 + 1.4)\,\Omega - 8\,V + V_a$$

$$V_a - V_c = 11\,V$$

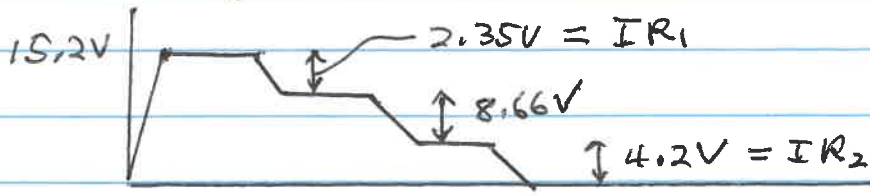
$I R_1 = \frac{40}{17} V = 2.35V$
$I R_2 = \frac{72}{17} V = 4.2V$

c) Terminal voltage of bottom battery (along current direction)

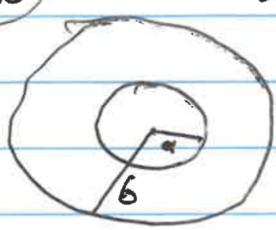
$$V_{bottom} = -8V - 1.4\Omega \frac{8}{17} A$$

$$= -8.66V$$

Start voltage to the right of the top emf



25.60



a) Material with resistivity η between two radii. Use

$$\eta J = E$$

⇒ note J and E vary with radius r.

$$J = I/4\pi r^2$$

⇒ I independent of r

$$E = \eta \frac{I}{4\pi r^2}$$

Find the potential drop

$$V = \int_a^b E\,dr = \frac{\eta I}{4\pi} \int_a^b dr \frac{1}{r^2}$$

$$= \frac{\eta I}{4\pi} \left. -\frac{1}{r} \right|_a^b = \frac{\eta I}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$R = \frac{\eta}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Another approach: know for a wire $R = \frac{\eta L}{A}$ with L the length and A the area. Consider dR

the resistance of a short radial length dr
$dR = \frac{\rho\, dr}{4\pi r^2}$. Integrate from "a" to "b" to give R.

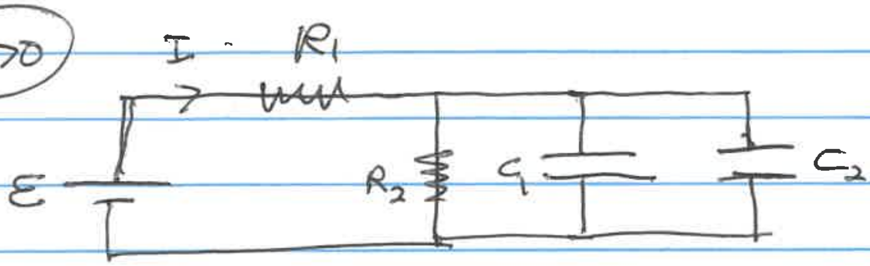
b) $J = \frac{I}{4\pi r^2} = \frac{4\pi}{\rho} \frac{1}{\left(\frac{1}{a} - \frac{1}{b}\right)} \frac{1}{4\pi r^2} V$

$= \frac{V}{\rho\left(\frac{1}{a}-\frac{1}{b}\right)} \frac{1}{r^2}$

c) For $a \simeq b$

$$R = \frac{\rho}{4\pi} \frac{b-a}{ab} = \frac{\rho\, L}{4\pi a^2} = \frac{\rho L}{A}$$

(25,70)



$\varepsilon = 72\,V$
$R_2 = 2\,\Omega$
$C_1 = 3\,\mu F$
$C_2 = 6\,\mu F$

At late time, $Q_1 = 18\,\mu C$

a) what is the charge on C_2 at late time?

$$V_1 = \frac{Q_1}{C_1} = V_2 = \frac{Q_2}{C_2}$$

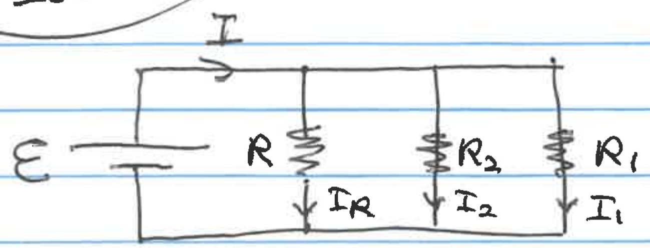
$$Q_2 = Q_1 \frac{C_2}{C_1} = 18\,\mu C\, \frac{6}{3} = 36\,\mu C$$

b) Potential drop across R_2 is $\frac{Q_1}{C_1} = \frac{18\,\mu C}{3\,\mu F} = 6\,V = I R_2 \Rightarrow I = 3\,A$

loop rule: $\varepsilon - I R_1 - I R_2 = 0$

$$R_1 = \frac{\varepsilon - I R_2}{I} = \frac{(72-6)\,V}{3\,A} = 22\,\Omega$$

26-20



$R = 10\Omega$
$I_1 = 2A$
$I = 3.5A$

$$P_1 = I_1^2 R_1 = 15W$$

$$R_1 = \frac{15W}{4A^2} = \frac{15}{4}\Omega$$

potential drops across R_1, R and R_2 are equal

$$I_1 R_1 = I_2 R_2 = I_R R$$

$$I_R = \frac{I_1 R_1}{R} = \frac{2A\ \frac{15}{4}\Omega}{10\Omega}$$

$$= \frac{3}{4}A$$

From point rules know

$$I = I_1 + I_2 + I_R$$

$$I_2 = I - I_1 - I_R = 3.5A - 2A - \frac{3}{4}A$$

$$= \frac{3}{4}A$$

$$R_2 = \frac{I_1 R_1}{I_2} = \frac{2A\ \frac{15}{4}\Omega}{\frac{3}{4}A} = 10\Omega$$

b) What is ε? Loop rule for big loop,

$$\varepsilon - I_1 R_1 = 0$$

$$\varepsilon = 2A\ \frac{15}{4}\Omega = \frac{15}{2}V$$

c) $I_2 = \frac{3}{4}A$, $I_R = \frac{3}{4}A$

d) Total dissipation in the resistors

$$P = P_1 + P_2 + P_R = 15W + \frac{9}{16}10W + \frac{9}{16}10W$$

$$= (15 + \frac{90}{8})W = 26.25W$$

power from $\mathcal{E}$ is $I\mathcal{E}$

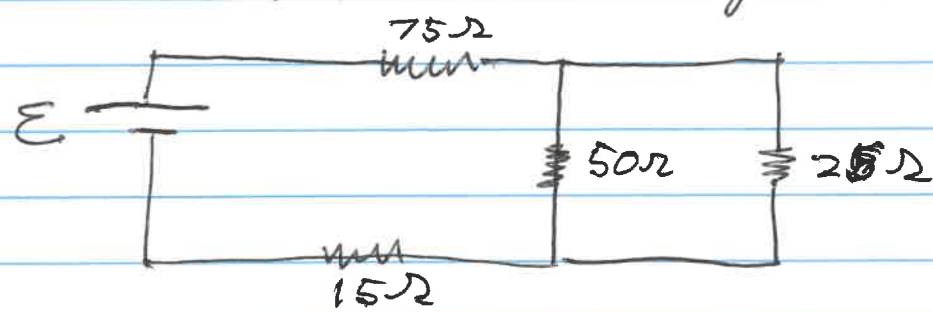
$$I\mathcal{E} = 3.5A\ \frac{15}{2}V = 26.25$$

$\Rightarrow$ energy conserved

26.47

a) just after the switch is closed there is no charge on any of the capacitors and no potential drop.

$\Rightarrow$ the circuit simplifies to



$\mathcal{E} = 100V$

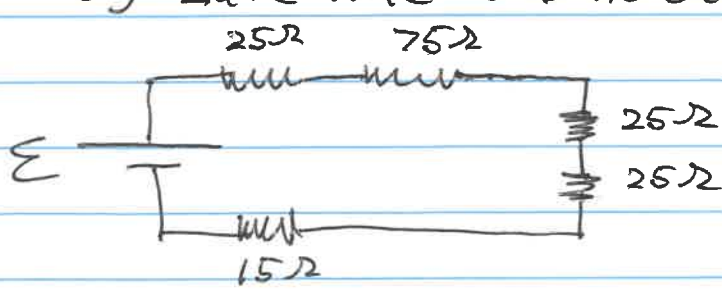
The right two resistors are in parallel

$$\Rightarrow \frac{1}{R} = \left(\frac{1}{50} + \frac{1}{25}\right)\frac{1}{\Omega} = \frac{3}{50}\frac{1}{\Omega}$$

$$R = \frac{50}{3}\Omega$$

$$I = \frac{100V}{\left(75 + 15 + \frac{50}{3}\right)\Omega} = 0.94\,A$$

b) Late time $\Rightarrow$ no current through capacitors



loop rule with resistors in series

$$R = 165\Omega$$

$$I = \frac{100V}{165\Omega} = 0.61\,A$$