Homework 9 Solutions

Q 30.15

When a steady current through an inductor is interrupted by opening a switch, an arc forms at the switch. An inductor with current stores magnetic energy. When the switch opens the circuit tries to maintain the current by arcing since the induced emf produced by the inductor becomes very large

$$\varepsilon_L = -L\frac{dI}{dt} = -L\frac{\Delta I}{\Delta t}$$

For small $\Delta t \Rightarrow \varepsilon_L$ is large and causes the air to break down, forming the arc. The magnetic energy is dissipated in the arc.

30.10

$L = 0.26\ H$

$V_a - V_b = 1.04\ V$

$= -L\frac{dI}{dt}$

$$\frac{dI}{dt} = -\frac{1.04\ V}{0.26\ H}$$

$= -4\ A/s \Rightarrow$ decreasing

b) $t = 0,\ I_0 = 12\ A$

$$I = I_0 + \frac{dI}{dt}t = 12A - 4\frac{A}{s}\,2s$$

$$= 4A$$

(30.16) Toroidal solenoid, N = # of turns = 300
mean radius $r = 0.12\ m$, A = area = $4\times10^{-4}\ m^2$,
$I = 5A$

a) $B = \frac{\mu_0 I N}{2\pi r} = 4\pi\times10^{-7}\frac{Tm}{A}\ \frac{5A\ 300}{2\pi(0.12m)}$

$= \frac{3\times10^{-3}}{1.2}\ T = 2.5\times10^{-3}\ T$

b) Self-inductance

$L = \frac{N\Phi}{I} = N^2\left(\frac{\mu_0}{2\pi r}\right)A$

$= 9\times10^4\ \frac{4\pi\times10^{-7}\ Tm}{2\pi(0.12m)\ A}\ 4\times10^{-4}\ m^2$

$= \frac{7.2}{1.2}\times10^{-5}\ H = 6\times10^{-5}\ H$

c) Energy stored

$W_L = \frac{1}{2}LI^2 = \frac{6\times10^{-5}}{2}\ 25\ J$

$= 7.5\times10^{-4}\ J$

d) magnetic energy density

$u_B = \frac{B^2}{2\mu_0} = \frac{6.25\times10^{-6}\ T^2 A}{2(4\pi)\times10^{-7}\ T\ m}$

$= \frac{6.25}{0.8\pi}\ \frac{J}{m^3} = 2.5\ \frac{J}{m^3}$

e) $\frac{W_L}{Vol} = \frac{7.5\times10^{-4}\ J}{4\times10^{-4}\ m^2\ 2\pi\ 0.12\ m} = \frac{7.5}{8\pi\ 0.12}\ \frac{J}{m^3}$

$= 2.5\ \frac{J}{m^3}$

30.34 $C = 7.5\,nF$, $V_C = 12\,V$

$\Rightarrow$ connected to coil

$\tau = 8.6 \times 10^{-5}\,s$

$$\omega^2 = \frac{1}{LC}, \quad \omega = \frac{2\pi}{\tau}$$

$$L = \frac{\tau^2}{C\,4\pi^2} = \frac{(8.6)^2 \times 10^{-10}}{7.5 \times 10^{-9}\,F\;4\pi^2}$$

$$= \frac{(4.3)^2}{0.75\,\pi^2} \times 10^{-2}\,H = 2.5 \times 10^{-2}\,H$$

b) Maximum charge on C is same as the initial charge since energy is conserved and passes back and forth between the inductor and capacitor.

$$V_c = \frac{Q}{C}$$

$$Q = V_c C = 12\,V\; 7.5 \times 10^{-9}\,F$$

$$= 9 \times 10^{-8}\,C$$

c) Total energy

$$W_{TOT} = \frac{1}{2} C V^2$$

$$= \frac{1}{2}\, 7.5 \times 10^{-9}\,F\; 144\,V^2$$

$$= 5.4 \times 10^{-7}\,J$$

d) Maximum current $\Rightarrow$ when all energy in L

$$W_{TOT} = \frac{1}{2} L I^2$$

$$I = \left(\frac{2\,W_{TOT}}{L}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2\,(5.4 \times 10^{-7})\,J}{2.5 \times 10^{-2}\,H}\right)^{\frac{1}{2}} = 6.6 \times 10^{-3}\,A$$

(30.45)

Solar magnetic energy

$$B = 0.4\,T$$

$$\rho = 3\times10^{-4}\,\frac{kg}{m^3}$$

$$u_B = \frac{B^2}{2\mu_0} = \frac{0.16\,T^2\,A}{(2)4\pi\times10^{-7}\,Tm}$$

$$= \frac{160}{8\pi}\times10^{4}\,\frac{J}{m^3} = 6.4\times10^4\,\frac{J}{m^3}$$

In $1\,m^3 \Rightarrow m = 3\times10^{-4}\,kg$

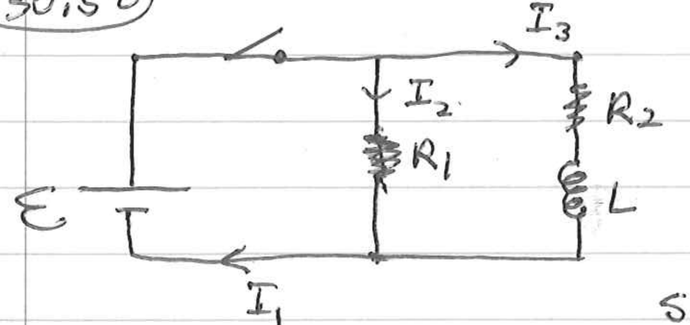
$$U_B = 6.4\times10^4\,J$$

$$\tfrac{1}{2}mv^2 = U_B$$

$$v = \left(\frac{12.8\times10^4\,J}{3\times10^{-4}\,kg}\right)^{\frac{1}{2}} = 2.1\times10^4\,\frac{m}{s}$$

$\Rightarrow$ less than escape speed.

(30.50)



$L = 0.3\,H$
$R_1 = 12\,\Omega$
$R_2 = 16\,\Omega$
$\mathcal{E} = 96\,V$

switch closed at $t=0$

a) Find I_1, I_2, I_3 just after switch closed

$I_3 = 0 \Rightarrow$ no time for magnetic energy to build up in L

Loop rule left loop,

$$\mathcal{E} - I_2R_1 = 0 \qquad I_2 = I_1 = \frac{96\,V}{12\,\Omega}$$

$$= 8\,A$$

b) After a long time $\frac{d}{dt} I_3 = 0 \Rightarrow \mathcal{E}_L = 0$

$I_2 = 8A$ as before

Loop rule large loop,

$$\mathcal{E} - I_3 R_2 = 0 \Rightarrow I_3 = \frac{96\ V}{16\ \Omega}$$

$$= 6\ A$$

$$I_1 = I_2 + I_3 = 14A$$

c) Outer loop is an R-L circuit

$$\tau = \text{time constant} = \frac{L}{R} = \frac{0.3\ H}{16\ \Omega}$$

$$= 1.9 \times 10^{-2}\ s$$

current I_3

$$I_3 = I_{3f}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$0.5 = \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$0.5 = e^{-t/\tau} \Rightarrow e^{\frac{t}{\tau}} = 2$$

$$\frac{t}{\tau} = \ln(2)$$

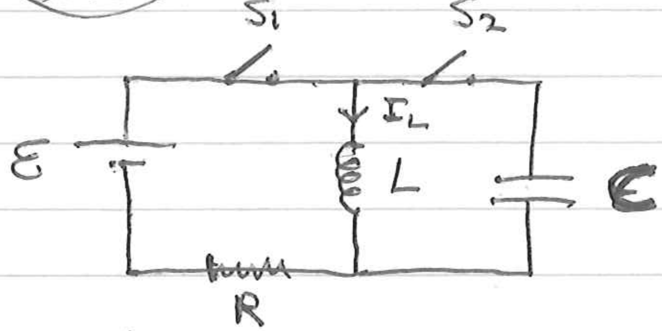
$$t = 1.9 \times 10^{-2}\ s \ln(2)$$

$$= 1.3 \times 10^{-2}\ s$$

d) $I_2 = 8A$

$$I_1 = 8A + 3A = 11\ A$$

30.60



L = 2 mH

C = 5 μF

With S_1 closed have steady current

$I_L = 3.5\,A$

Close S_2 and open S_1 at same time

a) RH loop has no resistor

$\Rightarrow$ energy will pass back and forth between inductor and capacitor

$$W_L = \frac{1}{2} L I^2 = \frac{1}{2} C V_c^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$Q = (LC)^{1/2}\, I$$

$$= \left(2 \times 10^{-3} H \; 5 \times 10^{-6} F\right)^{\frac{1}{2}} \; 3.5 A$$

$$= 3.5 \times 10^{-4}\, C$$

b) All the energy is in C so I in the inductor is zero

$$I_L = 0$$