Equation Sheet Physics 606

Maxwell's Equations

$$\nabla \times \underset{\sim}{H} = \underset{\sim}{J} + \frac{\partial}{\partial t} \underset{\sim}{D} \qquad \underset{\sim}{D} = \epsilon \underset{\sim}{E} = \epsilon_0 \underset{\sim}{E} + \underset{\sim}{P}$$

$$\nabla \times \underset{\sim}{E} + \frac{\partial}{\partial t} \underset{\sim}{B} = 0 \qquad \underset{\sim}{B} = \mu_0 (\underset{\sim}{H} + \underset{\sim}{M}) = \mu \underset{\sim}{H}$$

$$\nabla \cdot \underset{\sim}{B} = 0 \qquad \underset{\sim}{B} = \nabla \times \underset{\sim}{A}, \quad \underset{\sim}{E} = -\frac{\partial}{\partial t}\underset{\sim}{A} - \nabla \varphi$$

$$\nabla \cdot \underset{\sim}{D} = \rho \qquad \epsilon = \epsilon_0 (1 + \chi_e), \quad \rho_p = -\nabla \cdot \underset{\sim}{P}$$

$$\underset{\sim}{P} = \epsilon_0 \chi_e \underset{\sim}{E}$$

Math eqns

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 G(\underset{\sim}{x}, \underset{\sim}{x}') = -4\pi\, \delta(\underset{\sim}{x} - \underset{\sim}{x}') , \quad \delta(\underset{\sim}{x}) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} d\underset{\sim}{k}\, e^{i \underset{\sim}{k}\cdot \underset{\sim}{x}}$$

$$G(\underset{\sim}{x}, \underset{\sim}{x}') = \frac{1}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \qquad \int_{-\infty}^{\infty} dx\, \delta[f(x)] = \frac{1}{|f'(0)|}$$

Bessel Eqn: $\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) = 0$

$$J_\nu(x) \sim x^\nu (\text{small } x) \sim \frac{1}{x^{1/2}} \cos\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) (\text{large } x)$$

$$N_\nu(x) \sim x^{-\nu} (\text{small } x) \sim \frac{1}{x^{1/2}} \sin\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) (\text{large } x)$$

$$\sim \ln x \ (\nu = 0)$$

$$H_\nu^{(1)}(x) = J_\nu + i N_\nu , \quad H_\nu^{(2)}(x) = J_\nu - i N_\nu$$

Mod Bessel Eqn: $\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \left(1 + \frac{\nu^2}{x^2}\right) = 0$

$$I_\nu(x) \sim x^\nu (\text{small } x) \sim \frac{1}{x^{1/2}} e^{x} (\text{large } x)$$

$$K_\nu(x) \sim x^{-\nu} (\text{small } x) \sim \frac{1}{x^{1/2}} e^{-x} (\text{large } x)$$

$$\sim \ln x \ (\nu = 0)$$

$$\int_0^a d\rho\, \rho\, J_\nu^2\left(x_{\nu n} \frac{\rho}{a}\right) = \frac{a^2}{2} J_{\nu+1}^2 (x_{\nu n})$$

Legendre Eqn: $\frac{d}{dx}(1 - x^2)\frac{d}{dx} + \left[\ell(\ell+1) - \frac{m^2}{1-x^2}\right] = 0$

$\int d\Omega\, |Y_{\ell m}|^2 = 1$, $Y_{\ell m} = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\; P_\ell^m(\cos\theta)\, e^{im\varphi}$

Waves

$\mathbf{S} = \mathbf{E}\times\mathbf{H}$ $\qquad \Box^2 G = \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G = -4\pi\delta(\mathbf{x}-\mathbf{x}') \otimes \delta(t-t')$

$G = \frac{1}{|\mathbf{x}-\mathbf{x}'|}\,\delta\!\left(t' - \left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)\right)$

$\mathbf{P} = \frac{1}{c^2}\mathbf{E}\times\mathbf{H}$

wave eqn: $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix} = 0$

Electrostatics

charge: $\Phi(\mathbf{x}) = \frac{q}{4\pi\epsilon_0|\mathbf{x}|}$ $\qquad \mathbf{F} = q\mathbf{E}$ $\qquad u_E = \frac{1}{2}\epsilon E^2$

$\mathbf{E} = -\nabla\Phi$

dipole: $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\mathbf{x}}{|\mathbf{x}|^3}$ $\qquad \mathbf{p} = \int d\mathbf{x}\, \rho(\mathbf{x})\, \mathbf{x}$

surface charge: $E = \frac{\sigma}{2\epsilon_0}$ $\qquad U_q = q\Phi$, $U_p = -\mathbf{p}\cdot\mathbf{E}$

magnetostatics

$d\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{d\mathbf{l}\times\mathbf{x}}{|\mathbf{x}|^3}$ $\qquad$ wire: $B = \frac{\mu_0 I}{2\pi\rho}$

$\mathbf{B} = \nabla\times\mathbf{A}$

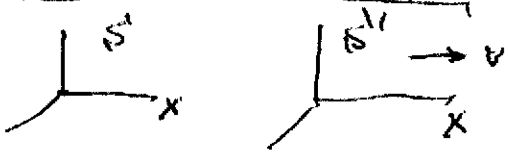
$d\mathbf{F} = I\, d\mathbf{l}\times\mathbf{B}$ $\qquad \mathbf{A} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{x}}{|\mathbf{x}|^3}$ mag. dipole

$U_m = -\mathbf{m}\cdot\mathbf{B}$ $\qquad \mathbf{m} = -\frac{I}{2}\oint d\mathbf{l}\times\mathbf{x}$

$u_B = \frac{1}{2\mu_0}B^2$ $\qquad m = IA$

Special Relativity

$\beta = v/c$ $\quad \gamma = 1/(1-\beta^2)^{\frac{1}{2}}$



$$\begin{pmatrix} x^{0\prime}\\ x^{1\prime}\\ x^{2\prime}\\ x^{3\prime}\end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix} x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}$$

4-vectors: $(ct, \mathbf{x})$, $\left(\frac{\mathcal{E}}{c}, \mathbf{p}\right)$, $\left(\frac{\omega}{c}, \mathbf{k}\right)$, $(c\rho, \mathbf{J})$

$\left(\frac{\Phi}{c^2}, \mathbf{A}\right)$ $\qquad \mathcal{E} = \gamma mc^2$, $\mathbf{p} = \gamma m\mathbf{u}$

velocity: $u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$, $\quad \mathbf{u}_\perp = \frac{\mathbf{u}_\perp}{\gamma_v\left(1 + \frac{v u_x'}{c^2}\right)}$

fields: $E_\parallel' = E_\parallel$, $B_\parallel' = B_\parallel$

$\frac{1}{c}\mathbf{E}_\perp' = \gamma\left(\frac{1}{c}\mathbf{E}_\perp + \boldsymbol{\beta}\times\mathbf{B}_\perp\right)$, $\quad \mathbf{B}_\perp' = \gamma\left(\mathbf{B}_\perp - \boldsymbol{\beta}\times\mathbf{E}_\perp\frac{1}{c}\right)$