Physics 762 Homework #1 Spring '24
Dr. Drake

1. Consider the equations for a nonlinear sound wave in a plasma with no magnetic field. Take the limit in which $k \ll k_D$ so that dispersive terms can be neglected but include a constant viscosity term in the momentum equation. Take the system to be isothermal.

   (a) Normalize the equations to get rid of all constants.

   (b) Transform to a frame of reference moving with the sound speed and develop an ordering in which dissipation balances wave steepening and the nonlinearity is weak.

   (c) Using this ordering derive a nonlinear equation for the system. This is Burgers equation and is given as follows:

   $$\frac{\partial n}{\partial t} + n\frac{\partial n}{\partial x} - \frac{1}{2}\frac{\partial^2 n}{\partial x^2} = 0 \tag{1}$$

   (d) Assume that this equation has a steady state solution which propagates at some velocity v_0. Derive this solution and show that it takes the form of a shock where the density approaches different values on either side of the shock. Find a relationship between the velocity of the shock and the density on either side of the shock.
   Hint: Integrate the steady state equation once and show that there are two solutions for the density, n_+ and n_-, on either side the the shock where $\partial n/\partial x \to 0$. Write the equation in terms of n_+ and n_- and integrate the equation again to obtain the final solution for $n(x)$.

   (e) Solve explicitly for the width of the shock in dimensional units.