HWK #3 Solutions

Unmagnetized Shock

jump conditions $\Rightarrow \underset{\sim}{V}_{t2} = 0$ since $\underset{\sim}{B}_t = 0$

$[nV] = 0$ continuity $\Rightarrow n_1 v_1 = n_2 v_2$

$$\left[\tfrac{1}{2} m n V^3 + \frac{\Gamma}{\Gamma-1} PV\right] = 0 \quad \text{energy}$$

$$\left[mnV^2 + P\right] = 0 \quad \text{pressure}$$

$$P_2 + \frac{m n_2^2 V_2^2}{n_2} = P_1 + m n_1 V_1^2$$

$n_1 v_1 = n_2 v_2$

$$P_2 + \frac{m n_1^2 v_1^2}{n_2} = P_1 + m n_1 v_1^2$$

(1) $$P_2 + \frac{m n_1 v_1^2}{r} = P_1 + m n_1 v_1^2 \quad \Rightarrow r = \frac{n_2}{n_1}$$

$$\frac{1}{2} \frac{m n_2^3 V_2^3}{n_2^2} + \frac{\Gamma}{\Gamma-1} \frac{P_2 V_2 n_2}{n_2} = \frac{1}{2} m n_1 V_1^3 + \frac{\Gamma}{\Gamma-1} P_1 V_1$$

$$\frac{1}{2} m \frac{n_1^3 v_1^{\cancel{3}\,2}}{n_2^2} + \frac{\Gamma}{\Gamma-1} \frac{P_2 n_1 \cancel{v_1}}{n_2} = \frac{1}{2} m n_1 v_1^{\cancel{3}\,2} + \frac{\Gamma}{\Gamma-1} P_1$$

(2) $$\frac{1}{2} m \frac{n_1 v_1^2}{r^2} + \frac{\Gamma}{\Gamma-1} \frac{P_2}{r} = \frac{1}{2} m n_1 v_1^2 + \frac{\Gamma}{\Gamma-1} P_1$$

Insert P_2 from 1 into 2

$$\frac{\frac{1}{2} m n_1 V_1^2}{r^2} + \frac{\Gamma}{\Gamma-1}\frac{1}{r}\left(P_1 + m n_1 V_1^2 - \frac{m n_1 V_1^2}{r}\right)$$

$$= \frac{1}{2} m n_1 V_1^2 + \frac{\Gamma}{\Gamma-1} P_1$$

$$\frac{1}{2} m n_1 V_1^2\left(1 - \frac{1}{r^2} - \frac{2\Gamma}{\Gamma-1}\frac{r}{r^2} + \frac{2\Gamma}{\Gamma-1}\frac{1}{r^2}\right)$$

$$+ \frac{\Gamma}{\Gamma-1} P_1\left(1 - \frac{1}{r}\right) = 0$$

Remove the $r = 1$ solution $\Rightarrow$ trivial solution

$$\frac{1}{2} m n_1 V_1^2\left((r+1)\cancel{(r-1)} - \frac{2\Gamma}{\Gamma-1}\cancel{(r-1)}\right) + \frac{\Gamma}{\Gamma-1} P_1 \cancel{(r-1)} r$$

$$= 0$$

$$\cancel{\frac{1}{2}} m n_1 V_1^2\left(r + 1 - \underbrace{\frac{2\Gamma}{\Gamma-1}}_{\alpha}\right) + \underbrace{\frac{2\Gamma}{\Gamma-1}}_{\alpha} P_1 r = 0$$

$$m n_1 V_1^2 (r + 1 - \alpha) + \alpha P_1 r = 0$$

$$r = \frac{(\alpha - 1) m n_1 V_1^2}{m n_1 V_1^2 + \alpha P_1} \qquad \alpha = \frac{2\Gamma}{\Gamma-1}$$

$$= \frac{(\Gamma+1) M_1^2}{2\Gamma + (\Gamma-1) M_1^2} \qquad M_1^2 = \frac{m n_1 V_1^2}{P_1}$$

High Mach # $\quad m n_1 V_1^2 \gg \alpha P_1$

$$r = \frac{2\Gamma}{\Gamma-1} - 1 = \frac{2\Gamma - (\Gamma-1)}{\Gamma-1} = \frac{\Gamma+1}{\Gamma-1}$$

$$\Gamma = \frac{5}{3} \Rightarrow r = 4$$

What fraction of incident kinetic energy flux $\frac{1}{2} m n_1 V_1^3$ is converted to thermal energy.

down stream: $n_2 = 4 n_1$

$$V_2 = \frac{1}{4} V_1$$

down streams kinetic energy flux

$$\frac{1}{2} m n_2 V_2^3 = \frac{1}{2} m 4 n_1 \frac{1}{4^3} V_1^3$$

$$= \frac{1}{2} m n_1 V_1^3 \frac{1}{16}$$

$\frac{15}{16}$ of the incident is converted to thermal.