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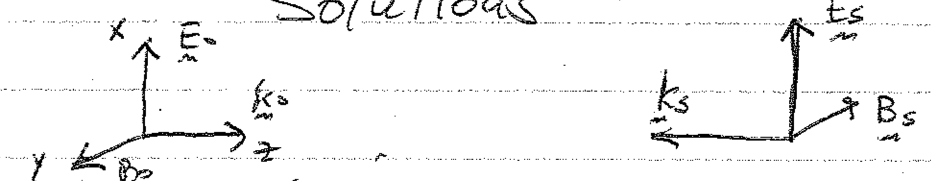
Physics 762 Hwk # 3
Solutions



(1) Derive equations for Brillouin scattering $\Rightarrow$ light scattering off of sound waves

$\Rightarrow$ include ion response to the low frequency wave.

<u>low frequency response</u> —

electrons

$$0 = -e n_0 \tilde{E}_z - T_e \frac{\partial}{\partial z}\tilde{n}_e - n_0 \frac{\partial}{\partial z}\left(\frac{e^2}{m_e}\underbrace{\frac{E_0 E_s}{\omega_0^2}}\right)$$

$$\sim e^{ik_0 z - i\omega_0 t} \otimes e^{ik_s z} e^{i\omega_s t}$$

ions

$$m_i \frac{\partial}{\partial t}\tilde{v}_{iz} = e\tilde{E}_z$$

$$\frac{\partial \tilde{n}_i}{\partial t} + n_0 \frac{\partial}{\partial z}\tilde{v}_{iz} = 0$$

$\Rightarrow$ note that ponderomotive force on ions is smaller than electrons because of larger ion mass

Take $\tilde{n}, \tilde{E}_z, \tilde{v}_{iz} \sim e^{ik_a z} e^{-i\omega_a t}$

$$k_0 + k_s = k_a, \quad \omega_0 = \omega_s + \omega_a$$

Poisson Eqn

$$i k_a \tilde{E}_z = 4\pi e\left(\tilde{n}_i - \tilde{n}_e\right)$$

$$0 = \cancel{+} n_0 e \tilde{E}_z \cancel{+} i k_a T_e \tilde{n}_e \cancel{+} i k_a n_0 \frac{e^2}{m_e}\frac{E_0 E_s}{\omega_0^2}$$

$$-i\omega_a m_i \tilde{V}_{zi} = e\tilde{E}_z$$

$$-\cancel{i}\omega_a \tilde{n}_i + n_0 \cancel{i} k_a \tilde{V}_{zi} = 0$$

$$\Rightarrow \tilde{n}_i = \frac{n_0 k_a}{\omega_a}\frac{e\tilde{E}_z}{-i\omega_a m_i}$$

$$\frac{i k_a \tilde{E}_z}{4\pi e} = \frac{n_0 k_a e\, i}{\omega_a^2 (\cancel{+}i) m_i}\tilde{E}_z - \tilde{n}_e$$

$$\frac{i k_a \tilde{E}_z}{4\pi e}\left(1 - \frac{\omega_{pi}^2}{\omega_a^2}\right) = -\tilde{n}_e$$

$$0 = \cancel{+} \frac{n_0 e}{T_e}\frac{\tilde{n}_e 4\pi e T_e}{\cancel{i} k_a^2\left(1 - \frac{\omega_{pi}^2}{\omega_a^2}\right)} + \cancel{i k_a} T_e \tilde{n}_e$$

$$+ \cancel{i k_a} n_0 \frac{e^2}{m_e}\frac{E_0 E_s}{\omega_0^2}$$

$$0 = \cancel{T_e}\frac{\tilde{n}_e}{n_0}\left(1 + \frac{k_{De}^2}{k_a^2}\frac{1}{1 - \frac{\omega_{pi}^2}{\omega_a^2}}\right) + \frac{e^2}{m_e}\frac{E_0 E_s}{T_e \omega_0^2}$$

$$0 = \frac{\tilde{n}_e}{n_0}\left(1 - \frac{\omega_{pi}^2}{\omega_a^2} + \frac{k_{De}^2}{k_a^2}\right) + \left(1 - \frac{\omega_{pi}^2}{\omega_a^2}\right)\frac{e^2}{m_e}\frac{E_0 E_s}{\omega_0^2 T_e}$$

$$\boxed{\frac{\tilde{n}_e}{n_0}\left(1-\frac{\omega_{pi}^2}{\omega_a^2}+\frac{k_{De}^2}{k_a^2}\right) = -\left(1-\frac{\omega_{pi}^2}{\omega^2}\right)\frac{e^2}{m_e T_e}\frac{E_0 E_s}{\omega_0^2}}$$

<u>Scattered Wave</u> $\Rightarrow$ same as Raman

$$\boxed{\left(k_s^2c^2+\omega_{pe}^2-\omega_s^2\right)E_s = -\omega_{pe}^2\frac{\tilde{n}_e}{n_0}E_0^*}$$

from Raman $\quad \omega_0 = \omega_s + \omega_a \quad \underset{\sim}{k_0} = \underset{\sim}{k_s} + \underset{\sim}{k_a}$

$$\left(k_s^2c^2+\omega_{pe}^2-\omega_s^2\right) \simeq 2\omega_0\left(\omega_a - \Delta\omega\right)$$

$$\Delta\omega = (2k_0 - k_a)c \qquad k_0 = -k_s + k_a$$

note: $\frac{\Delta\omega}{\omega_0} = \frac{(2k_0-k_a)\cancel{c}}{k_0\cancel{c}} \ll 1 \qquad k_s = k_a - k_0$

$$\Longrightarrow \quad k_a \simeq 2k_0$$

<u>Dispersion Relation</u>

$$\left(1-\frac{\omega_{pi}^2}{\omega^2}+\frac{k_{De}^2}{k_a^2}\right)2\omega_0\left(\omega_a-\Delta\omega\right) = \frac{\left(1-\frac{\omega_{pi}^2}{\omega_a^2}\right)\omega_{pe}^2\,e^2|E_0|^2}{m_e\omega_0^2 T_e}$$

$$\left(1-\frac{\omega_{pi}^2}{\omega^2}+\frac{k_{De}^2}{k_a^2}\right)\left(\omega_a-\Delta\omega\right) = \left(1-\frac{\omega_{pi}^2}{\omega_a^2}\right)\frac{\cancel{\omega_{pi}^2}\,|V_0|^2\,\cancel{m_e}}{2\omega_0\cancel{T_e}}$$

k_{De}^2

$$\boxed{\left(1 - \frac{\omega_{pi}^2}{\omega_a^2} + \frac{k_{De}^2}{k_a^2}\right)(\omega_a - \Delta\omega) = \left(1 - \frac{\omega_{pi}^2}{\omega_a^2}\right) \frac{k_{De}^2 |v_0|^2}{2\omega_0}}$$

with $k_a \simeq 2k_0$ and $\Delta\omega$ a free parameter (replacing k_0)

<u>Weak growth</u>

Take $k_a \ll k_{De}$

$\omega_a \simeq k_a c_s \qquad c_s = \sqrt{\frac{T_e}{m_i}}$

$$(\omega_a^2 - k^2 c_s^2)(\omega_a - \Delta\omega) = -\omega_{pi}^2 k_a^2 \frac{|v_0|^2}{2\omega_0}$$

$$(\omega_a - kc_s)(\omega_a - \Delta\omega) = -\omega_{pi}^2 \frac{\cancel{4k_0^2} |v_0|^2}{\cancel{4} k_0 c \; 2\cancel{k_0} c_s}$$

$$= -\omega_{pi}^2 \frac{|v_0|^2}{2 c c_s}$$

let $\Delta\omega = kc_s$ (max growth)

$\omega_a = kc_s + i\gamma$

$$\boxed{\gamma^2 = \omega_{pi}^2 \frac{|v_0|^2}{2 c c_s}}$$

Brillouin Instability

$$\boxed{\gamma \sim v_0}$$

Large growth rate but take

$\Delta\omega, k c_s < \omega < \omega_{pi}$

$$\cancel{\frac{k_{De}^2}{k_a^2}}\left(\omega_a - \cancel{\Delta\omega}\right) = -\frac{\omega_{pi}^2}{\omega_a^2}\,\frac{\cancel{k_{De}^2}\,|V_0|^2}{\cancel{4}\cancel{k_0 c}}\,\cancel{4k_0^2}$$

$$\omega_a^2\left(\omega_a - \cancel{\Delta\omega}\right) = -\omega_{pi}^2\, 2k_0\,\frac{|V_0|^2}{c}$$

$$\omega_a = \left(\frac{2k_0\,\omega_{pi}^2\,|V_0|^2}{c}\right)^{\frac{1}{3}} e^{i\frac{\pi}{3}}$$

$$\boxed{\omega_a \sim V_0^{2/3}}$$

$\Rightarrow$ weaker scaling with V_0