

Math 420, Spring 2012

HIV/AIDS Modeling Interim Project 1

due Wednesday April 4

In class we discussed the two-group SIR model with growth:

$$\begin{aligned}dS_1/dt &= q_1 - p_{11}S_1\mathcal{I}_1 - p_{12}S_1\mathcal{I}_2 \\dS_2/dt &= q_2 - p_{21}S_2\mathcal{I}_1 - p_{22}S_2\mathcal{I}_2 \\d\mathcal{I}_1/dt &= p_{11}S_1\mathcal{I}_1 + p_{12}S_1\mathcal{I}_2 - r_1\mathcal{I}_1 \\d\mathcal{I}_2/dt &= p_{21}S_2\mathcal{I}_1 + p_{22}S_2\mathcal{I}_2 - r_2\mathcal{I}_2 \\dR_1/dt &= r_1\mathcal{I}_1 \\dR_2/dt &= r_2\mathcal{I}_2\end{aligned}$$

In this model, each at-risk person is either in group 1 or group 2, and always remains in that group. If there are n groups, we can write the equations more concisely as follows, where P is an $n \times n$ matrix of parameters, $\mathcal{I}$ is a column vector with entries $\mathcal{I}_1, \dots, \mathcal{I}_n$, and $(P\mathcal{I})_j$ denotes the j th coordinate of the vector $P\mathcal{I}$:

$$\begin{aligned}dS_j/dt &= q_j - (P\mathcal{I})_j S_j \\d\mathcal{I}_j/dt &= (P\mathcal{I})_j S_j - r_j \mathcal{I}_j \\dR_j/dt &= r_j \mathcal{I}_j,\end{aligned}$$

for each $j = 1, \dots, n$.

For this assignment, consider the case $q_1 = q_2 = r_1 = r_2 = 0$, a 2-group SI model. In this case, the populations $N_1 = S_1 + \mathcal{I}_1$ and $N_2 = S_2 + \mathcal{I}_2$ are constant over time. Consider group 1 to be people with high-risk behavior for transmitting HIV and group 2 to be people with lower-risk behavior.

1. Think about what are reasonable values of the parameters $p_{11}, p_{12}, p_{21}, p_{22}$. Should they be related to each other in some way, e.g., should $p_{12} = p_{21}$? Which parameters should definitely be larger than which other parameters?
2. Simulate the model for different values of the parameters and the initial populations. Look at when the rate of new infections peaks for each group, i.e., when $d\mathcal{I}_1/dt$ peaks versus when $d\mathcal{I}_2/dt$ peaks. Can you deduce any general rules about how the parameters affect the time difference between the two peaks?