Math 246, Professor David Levermore
Group Work Questions for Discussion
Monday, 19 February 2018

First Set of Group Work Questions [5]

These questions are based upon Problem 2b of Exam 1, which was to find an explicit solution and give its interval of definition for the initial-value problem

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{z^2 - 9}{6}\, e^t\,, \qquad z(0) = 2\,.$$

Based upon the posted solution of this problem, address the following questions.

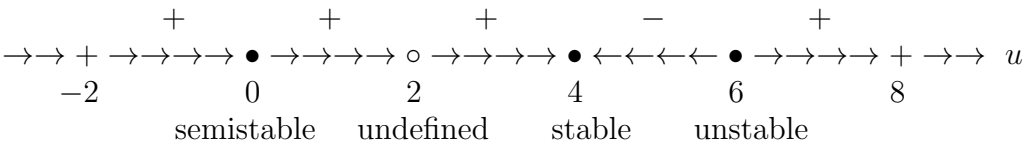
(1) How does the solution $z(t)$ behave as $t \to \infty$?
(2) How does the solution $z(t)$ behave as $t \to -\infty$?
(3) How do the answers change if the initial condition is $z(3) = 2$?
(4) How do the answers change if the initial condition is $z(0) = 5$?
(5) How do the answers change if the initial condition is $z(0) = -5$?

Second Set of Group Work Questions [5]

These questions are based upon Problem 3 of Exam 1, which was about the phase-line portrait of the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{u^2(4-u)(6-u)}{(1+u^2)(2-u)^2}\,.$$

Most students obtained the following correct phase-line portait.



However, many had difficulty using this phase-line to see how nonstationary solutions behave. Based upon this phase-line portrait and upon the equation, address the following questions.

(1) How does $u(t)$ behave as t increases when $u(0)$ is in the interval $(-\infty, 0)$? Does the solution exist as $t \to \infty$ or does either $u(t)$ or $u'(t)$ blow-up at a finite time t_*?
(2) How does $u(t)$ behave as t increases when $u(0)$ is in the interval $(0, 2)$? Does the solution exist as $t \to \infty$ or does either $u(t)$ or $u'(t)$ blow-up at a finite time t_*?
(3) How does $u(t)$ behave as t increases when $u(0)$ is in the interval $(2, 4)$? Does the solution exist as $t \to \infty$ or does either $u(t)$ or $u'(t)$ blow-up at a finite time t_*?
(4) How does $u(t)$ behave as t increases when $u(0)$ is in the interval $(4, 6)$? Does the solution exist as $t \to \infty$ or does either $u(t)$ or $u'(t)$ blow-up at a finite time t_*?
(5) How does $u(t)$ behave as t increases when $u(0)$ is in the interval $(6, \infty)$? Does the solution exist as $t \to \infty$ or does either $u(t)$ or $u'(t)$ blow-up at a finite time t_*?

Remark. Similar questions can be asked about how $u(t)$ behaves as t decreases when $u(0)$ is in each of the intervals $(-\infty, 0)$, $(0, 2)$, $(2, 4)$, $(4, 6)$, and $(6, \infty)$. Does the solution exist as $t \to -\infty$ or does either $u(t)$ or $u'(t)$ blow-up at a finite time t_*?