Quiz 9, Math 246, Professor David Levermore
Thursday, 12 April 2018

Your Name:

Discussion Instructor (circle one): Kilian Cooley Corry Bedwell Thien Ngo
Discussion Time (circle one): 8:00 9:00 10:00 11:00

No books, notes, calculators, or any electronic devices.
Show your reasoning for full credit. Good luck!

(1) [2] Transform the equation $y''' - y^2 y'' + (y')^3 - \cos(t+y) = e^y$ into a first-order system of ordinary differential equations.

(2) [3] Give the interval of definition for the solution of the initial-value problem

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \dfrac{4e^t}{25-t^2} & \dfrac{3}{t} \\ \dfrac{3}{t} & \dfrac{4e^t}{25-t^2} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}, \qquad \begin{pmatrix} u(-3) \\ v(-3) \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}.$$

(3) [5] Consider the vector-valued functions $\mathbf{x}_1(t) = \begin{pmatrix} e^t \\ -t^2 \end{pmatrix}$, $\mathbf{x}_2(t) = \begin{pmatrix} e^t \\ 1 \end{pmatrix}$.

(a) [2] Compute their Wronskian $\mathrm{Wr}[\mathbf{x}_1, \mathbf{x}_2](t)$.

(b) [3] Find $\mathbf{A}(t)$ such that $\mathbf{x}_1$, $\mathbf{x}_2$ is a fundamental set of solutions to $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$.