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Using Embodiment to Reason About Geometry in an Integrated Computational Thinking and Mathematics Activity

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Abstract: This work uses grounded and embodied cognition (GEC) perspectives to explore how young learners reason through computational thinking (CT) concepts and geometric angles in integrated mathematics and CT-enhanced robotics activities. Findings from video data reveal that different modalities (e.g., Sphero robot, Sphero.edu app, tape, a triangle path) enabled learners to reason through geometry. We highlight the importance of examining the complex contexts in which young learners learn and apply CT and mathematical knowledge. We show that embodied gestures can advance our understanding of young learners' reasoning about geometry, particularly in robotics activities requiring spatial reasoning abilities for navigation. Analyses support claims that geometry and spatial thinking skills in integrated mathematics, CT and, robotics environments for young learners can be expressed through embodied gestures. These findings have implications for instructional practices and the design of educational technologies and activities to support students' embodiment of concepts in ways that can support learning.

Introduction

Understanding and applying computational thinking (CT) skills is essential in today's technology-driven world. As Wing (2006) emphasized, CT skills are as fundamental as reading, writing, and arithmetic in shaping every child's analytical ability. Researchers have made significant efforts to integrate CT into classrooms, either by advocating for broader participation or by developing curricula that provide opportunities for students to learn and build foundational CT skills (National Research Council, 2010; Shute et al., 2017; Wing, 2006; Weintrop et al., 2022). CT integration across grades, contexts, and classrooms differs significantly based on activity and tools. Early-grade learners often have little prior experience in programming or advanced mathematical concepts due to limited exposure to tools and activities that combine these disciplines (Fofang et al., 2020). Identifying young learners' understanding of mathematics concepts through verbal explanations, written responses, or created artifacts during robotics learning activities can be challenging. By observing learners' interactions and embodied ways learners navigate space, move, and gesture, we can understand how their reasoning is expressed.

Reasoning through mathematical ideas and notation systems is a core focus in STEM education (e.g., Schoenfeld, 1992). Scholars have increasingly explored the role of grounded and embodied cognition (Barsalou, 2008; Glenberg & Robertson, 2000). For example, scholars have argued that we might understand mathematical ideas through metaphors involving movement or space, making abstract ideas more meaningful through the body's interaction with the world (Lakoff & Nunez, 2001). Research on embodied mathematics has explored how young children build foundational math skills by linking mathematical concepts with physical actions and sensory experiences (e.g., Butterworth, 1999).

In more complex, multimodal activities that integrate CT and mathematics, such as activities that involve robots, bringing an embodied perspective can help deepen our understanding of how learners reason with and about CT. Investigating whether and how embodied experiences facilitated by CT tools support students' mathematical reasoning and learning is essential. This integration goes beyond coding skills to show how programming robots supports embodied understanding of geometry. It uniquely combines CT, spatial reasoning, and physical interaction, helping young learners grasp abstract math concepts like angles through real-time, hands-on experience. This shift in focus to investigating mathematical reasoning in CT-enhanced robotics contexts is especially relevant to advance our understanding of how young learners learn complex topics such as geometry, a branch of mathematics concerned with generalizable properties of shape and space. Traditional mathematics curricula focus on lowering complexity in the presentation of the angles, primarily focusing on concrete concepts in geometry, such as identifying and naming shapes, recognizing their attributes (like sides and vertices), and understanding fundamental spatial relationships (Pier et al., 2019). As such, instruction often relies on manipulatives and visual aids rather than engaging students in the more advanced logical reasoning of values of an angle associated with abstract mathematics (Nathan & Walkington, 2017). Programming geometric shapes in robotics tasks requires students to use spatial reasoning skills to abstract and articulate concepts like angles, challenging learners to reason about space and shapes integrated into programming activities. Young learners can sometimes rely on embodied gestures to express spatial reasoning skills during robotics programming tasks.

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Investigating the role of embodied gestures in learning and programming geometric angles in robotics tasks for young learners could uncover how embodied experiences and spatial reasoning could enhance students' understanding of abstract mathematical concepts such as angles and CT skills.

The primary goal of this paper is to examine fourth-grade students' geometric and spatial thinking skills through an embodied lens, drawing on empirical data from activities in which learners program robots in integrated mathematics and CT tasks. More concretely, this work seeks to answer the following research question. *How do young learners without programming experience express their reasoning about geometric angles while programming a robot to navigate a triangle path?* We focus on the role and nature of students' embodied gestures as they program the Sphero robot to navigate geometric shapes and angles in CT tasks. We use frameworks from grounded and embodied cognition (GEC) to investigate how young learners reason about geometric shapes as they program the Sphero robot to navigate a triangle. This research is significant for extending the application of GEC frameworks to dynamic environments such as integrated mathematics and CT robotics contexts. This advances our knowledge of how bodily experiences through gestures and spatial interactions can facilitate complex reasoning processes, such as understanding and applying geometric concepts in dynamic environments. Insights from this work can inform the design of effective, evidence-based approaches to designing integrated CT and mathematics activities for early-grade learners, consequently shaping design strategies for both students' learning environments and teacher understanding of CT knowledge acquisition.

Previous work

Computational thinking in elementary grade classrooms

Papert's work made a groundbreaking contribution by demonstrating that computers can be powerful tools to support learning, especially for younger students (Papert, 1980). His work on Constructionism, alongside his collaborators, set the stage for what we now refer to as Computational Thinking (Wing, 2006). In response to the increasing recognition of CT as essential, a growing number of initiatives are aimed at integrating CT into K-12 classrooms (Weintrop et al., 2016, 2022). CT encompasses the concepts, practices, and skills necessary to solve problems using computational tools, including developing algorithms, breaking down complex problems, and debugging solutions (Grover & Pea, 2013; Shute et al., 2017). Research on CT has expanded significantly, with efforts to integrate CT into various disciplines (e.g., Barr & Stephenson, 2011; Wilensky et al., 2014) as well as research focused on supporting younger learners in engaging with CT concepts (Weintrop et al., 2022).

The past decade has witnessed a significant increase in initiatives to leverage computational tools as contexts for teaching foundational CT concepts (Shute et al., 2017; Weintrop et al., 2016), with some using mathematics as a context for embedding CT (e.g., weintrop et al, 2022). A key premise for integrating CT into mathematics is the mutually supportive relationship between CT and mathematics disciplinary content: CT can enhance understanding within mathematics while mathematics lends itself to relevant contexts for applying CT concepts and practices (e.g., Abelson & diSessa, 1986; Papert, 1972; Bih et al., 2020). This study builds on prior research that uses robotics to contextualize mathematical concepts and engage young learners in applying computational thinking practices. Scholars have argued that introducing robots into classrooms can support learners to develop CT skills, especially in the early grades. (e.g., E. Kazakoff & Bers, 2012; Mioduser, Levy, and Talis, 2009). As part of integrating constructionist and programming activities into early-grade classrooms, a good number of studies have cautioned about the importance of scaffolding these activities using developmentally appropriate tools and interventions in order to meet the cognitive expectations of learners at different stages of their development (e,g. Marina U Bers, 2005, 2010; Brennan and Resnick, 2012). Recent work in STEM education emphasizes that CT is not only a logical or symbolic process but also deeply tied to learners' physical interactions with tools and space. Embodied cognition frameworks suggest that thinking emerges through action, perception, and sensorimotor experiences, aligning well with CT activities that require physical programming of robots or navigating spatial problems. By integrating these perspectives, we can better understand how learners use gestures, movement, and physical enactment to reason through and express CT concepts.

This study extends prior work by using a *Grounded and Embodied Cognition* lens to examine how young learners use gestures and bodily movement to reason about geometric angles during robot programming. Unlike earlier studies that focus on coding outcomes or verbal reasoning, this work highlights dynamic depictive gestures as tools for thinking, showing how students embody both robot and path, to understand spatial relationships. This offers new insight into how embodiment supports abstract mathematical reasoning in CT-integrated tasks.

Reasoning about mathematics through embodiment

A growing body of research on grounded and embodied cognition (GEC) suggests that thinking and reasoning are closely linked to body-based processes, such as gestures and physical or imagined actions. For example, gestures

(Alibali & Nathan, 2012; Goldin-Meadow, 2020), real and simulated actions (Hostetter & Alibali, 2019), have been found to play a role in how we understand and process information. In mathematics, embodied cognition has offered potential insights into how learners can grasp highly abstract concepts and symbols. For example, theoretical frameworks, such as those by Abrahamson & Lindgren (2014) and Lakoff & Núñez (2000), suggest that even the most abstract forms of mathematical thinking may be grounded in bodily experiences. While grounded and embodied cognition (GEC) theories may vary, they share some core ideas. One key idea is that thinking and CT are not solely based on abstract, symbolic operations; instead, reasoning and computation can engage sensory and motor systems. Additionally, programming in robotics supports embodied learning by turning abstract math concepts into physical experiences. As students program movement, they use gestures or walk paths to reason about angles and distances, grounding mathematical ideas in action and sensory feedback.

Research has shown that, when planning or problem-solving without physical cues, the mind often uses mental simulations of past sensory and motor experiences, which play a crucial role in shaping reasoning and problem-solving (Barsalou, 2008). Some researchers have noticed that learners often naturally and spontaneously use gestures and body movements to help them think through and communicate mathematical ideas. For example, learners have been reported using their hands to show the size of an angle, tracing the shape of a graph in the air, or moving their bodies to illustrate a transformation like rotation or symmetry(e.g., Alibali & Nathan, 2012). These physical actions can serve as tools for exploring mathematical concepts, making abstract ideas more concrete, and expressing reasoning directly and intuitively.

Furthermore, most embodied mathematics research has focused on young learners' early mathematical development, examining how body-based actions and gestures can help them understand basic concepts (e.g., Butterworth, 1999). Our focus on geometry for early-grade learners in robotics tasks is important because geometry requires students to articulate logically sound, generalized truths about shapes and spatial relationships to program robots. Exploring young learners' geometric reasoning in dynamic robotics contexts could shed light on how embodiment and gestures can be explored to enhance our understanding of learning through practical experiences in integrated mathematics and CT contexts.

Dynamic gestures and spatial reasoning

Research suggests that compared to learners with lower spatial reasoning skills, learners with stronger spatial reasoning skills tend to use more dynamic gestures to help them convey complex ideas about how objects can move or change (Harel & Sowder, 2005; Pier et al., 2019). Real-time interactions afforded by dynamic robot behaviors can contribute to transforming abstract mathematics activities into concrete, felt experiences, deepening understanding through action. Some studies have shown that learners who used dynamic gestures were likelier to have correct insights and successfully produce correct proofs of mathematical concepts than those who did not (Göksun, Goldin-Meadow, Newcombe, & Shipley, 2013). Dynamic gestures are movements of the hands, arms, or body that simulate or represent motion, transformation, or changes in objects or spatial configurations. Dynamic gestures can engage cognitive processes in ways that are especially useful for generalizing mathematical reasoning. For example, studies have reported that when learners perform a gesture to simulate a movement, like rotating an imagined triangle, their brain generates a "motor program" that predicts how the triangle might change (e.g., Chu & Kita, 2011). This kind of prediction may help learners make inferences about future states or transformations of the shape or reason about the shape (Nathan & Walkington, 2017). This action-based thinking process can allow learners to predict potential outcomes, consequently supporting their reasoning about spatial properties essential for mathematical proofs (Hostetter & Alibali, 2019). Thus, an embodied approach to inference-making in spatial reasoning contexts can help learners form the generalizations needed for mathematical understanding, which young learners might find challenging to create without embodied cognitive support.

The data for this study was analyzed using a GEC framework, the Gesture as Simulated Actions (GSA) framework by Hostetter and Alibali (2019), to understand how learners' embodied actions and gestures contributed to the quality of their geometric reasoning as they programmed a Sphero robot to navigate an equilateral triangle. The Gesture as Simulated Actions (GSA) framework explains that gestures arise from mental simulations of actions or perceptual states, and their likelihood increases when these simulations are highly activated (Hostetter & Alibali, 2019). By examining connections between gestures and interactions with objects, this research seeks to identify meaningful links between embodied processes like gestures and geometric thinking and how these links facilitated learners' understanding of geometric shapes and angles in a robotics programming task.

Methods

The Robot.Math project

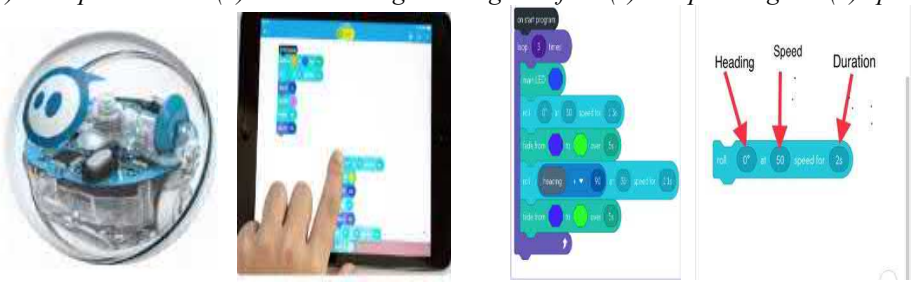
The Robot.Math project is developed through a research practitioner partnership (RPP). The Robot.Math curriculum was designed through collaborations between researchers working on the project, a technology specialist, an instructional coach, and 4th-grade mathematics teachers (Authors, 2021).

Robot.Math was designed to understand the feasibility of integrating CT in mathematics classrooms and how this cross-disciplinary integration can support student's understanding of CT while deepening their understanding of mathematical concepts. Each lesson includes materials for both students and teachers, with teacher-facing materials clearly defining the lesson objectives and outlining how the lesson aligns with the 4th-grade Common Core Mathematics Standards, followed by the district. The lessons also specify CT concepts students will explore and, where applicable, identify connections to the district's mandated mathematics curriculum.

The focal activity selected for this study is titled "Triangles" and involves modeling triangles to clean structures in an amusement park. This framing is built around the idea that triangles can strengthen and stabilize structures. Learners were tasked with programming Sphero robots to navigate an equilateral triangle taped on the floor. Learners were tasked to program the Sphero robot (see Figure 1a) using tablets (see Figure 1b). Sphero robots can be programmed using a block-based interface, which allows learners to build code by dragging and dropping visual blocks (see Figure 1c). Sphero programs are represented as block-based scripts (see Figure 1c) that can use basic programming elements such as loops and conditionals, as well as commands to regulate the Sphero's movement or actions (e.g., roll, strobe). The fundamental movement command for the Sphero is the roll command figure (1d), which can take inputs for heading, speed, and duration(time). The example shown on Figure. 1d is exemplified by a command that reads: "roll 0° at 50 speed for 2s."

Figure 1

(a) The Sphero Robot (b) iPad and Programming Interface (c) Sample Program (d) Sphero Roll Block



(a) (b) (c) (d)

Study design

Setting and participants

The study took place over two years in a 4th-grade mathematics classroom in an urban public school district in the Mid-Atlantic region of the United States. The school has a total enrollment of 255 students with the following racial breakdown: 45.1% Hispanic/Latino, 52.2% Black/African American, 2% White non-Hispanic, 0.8% Asian or Asian/Pacific Islander. This study involved focal students identified and selected for observation by the classroom teacher based on their ability to participate and express themselves in group activities. For this study, we share video data and field notes of one student's interactions from a group of three students (in one focal group) as they programmed the Sphero robots to navigate a triangle to ground their understanding of CT and mathematical concepts.

Data collection

The primary data sources for this study were videos collected during the Sphero.Math lessons and student artifacts, including images of codes written in notebooks and images of their programs. We collected close-up video footage using phones, iPads, head-mounted cameras worn by students, and stationary cameras positioned throughout the activities. This setup allowed us to capture data on student-to-student interactions and perspectives, focusing on students' interactions with the Sphero robot as they completed 14 lessons of integrated CT and mathematics tasks. For this particular study, the focus on one student allows for in-depth analysis of gestures and embodied reasoning, offering rich insights into how CT and geometry connect in action. While not generalizable, this approach surfaces detailed patterns that can inform future research and broader studies.

Data analysis

We used coarse-grained interaction analysis (IA) for data analysis. We provide analysis in the form of data captured in ways that provide opportunities for repeated examination, allowing for alternative interpretations

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supported by evidence through transcripts from recordings and observations (Sacks, 1995; Atkinson & Heritage, 1984). Our data analysis was conducted as follows. Initially, team members reviewed all recorded videos to identify and segment video clips with instances where students used embodied gestures to navigate tasks. This process was accompanied by group discussions to refine our observations (Jordan & Henderson, 1995). We anchored our insights in concrete evidence of embodied practices observed, recorded on field notes, and captured on video. One researcher documented descriptive and analytical memos based on transcripts from video segments where students used embodiment to complete tasks. We maintained flexibility in our analysis in order to uncover and explore forms of embodied gestures and actions that learners used during activities, leading to fresh perspectives on articulating and interpreting the data in ways to provide opportunities, "space and place for exploration and discovery" (Charmaz, 2006, p. 81–82). One researcher compiled all the memos and video links into a shared spreadsheet, including specific evidence of observations. This structure enabled the team to analyze within and across memos.

Finally, the research team analyzed these memos using the GSA framework (Hostetter & Alibali, 2019) to highlight instances where students used embodied gestures while programming the Sphero robot to navigate a triangle path. Video segments were selected through a review of all recorded sessions, focusing on moments when students were actively programming or discussing robot movement. Segments were flagged when students used their hands, arms, or body to simulate motion, direction, or spatial transformations. We narrowed our focus to analyze only gestures that reflected *dynamic depictive gestures*, defined by GEC frameworks as *hand or body movements that illustrate changes or transformations in shapes and spatial relationships* (Nathan et al., 2021, p.7). For example, three kinds of gestures were taken into consideration in our analysis: gestures that visually demonstrated properties that can be generalized, such as enlarging or shrinking (dilating) and slanting or distorting (skewing) shapes, providing us with insights as to how learners are reasoning about geometric shapes, gestures that visually depicted movement, such as turning, rotating, or walking the robot's path and, movements that corresponded with specific mathematical ideas (e.g., angles, direction changes).

Findings

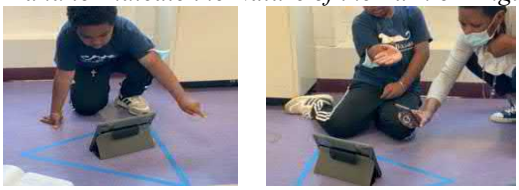
In our selected observations, we report instances when one student from our focal group used embodied gestures aligned with dynamic depictive gestures to reason about an equilateral triangle's geometric shapes and angles. Throughout the activity, the learner used dynamic depictive gestures in three instances: (a) swerving his palm and body to depict the turns (angle) the robot would make; (b) walking on the triangle path, making adjustments on the virtual protractor on the iPad, measuring the triangle's angles using a physical protractor, and (c) walking on the triangle with his right hand stretched out in front, without the virtual or physical protractor.

Swerving hand and body gestures to represent angles

First, we observed the learner swerving his palm and body to depict the turns (angle) the robot would make. The student begins by using hand-pointing gestures to illustrate his expectations for the robot's movement (see Figure 2a) and then swerves his palm and body (see Figure 2b) to indicate the turns (angles) the robot will make. He repeatedly used his palm to demonstrate how he anticipated the robot would move, emphasizing the angles with the movement of his hand.

Figure 2

(a) A Student Uses His Finger to Show the Direction in Which He Would Like to Program the Sphero to Move; (b) Student Swerves/Curves His Body and Hand to Indicate the Nature of the Turn or Angle the Robot Must Make



(a) (b)

The student's swerving palm gesture to indicate the turns (angles) exemplifies a dynamic depictive gesture, where his physical movements provide a visual change in direction (turn) on the triangle's path. In this case, the learner's gesture illustrates the angles the robot must turn to navigate one side of the equilateral triangle. The student connects imaginary abstract geometric ideas, like angles, by physically enacting these transformations to tangible, real-world movements (turns). This gesture visually demonstrates the learner's understanding of angles, allowing others to see how he is reasoning about the geometric path the robot had to navigate. This

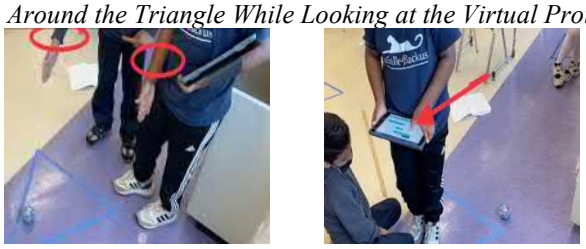
demonstrates how the student expresses geometric angle reasoning through physical movement. These gestures represent simulated actions, showing an internal visualization of the robot's turning path, a core notion in GSA's concept of dynamic depictive gestures.

Walking the triangle path with virtual protractor

We also observed a second instance when the student, supported by the virtual protractor on the iPad, embodied the robot and walked on the triangle path to mimic the robot's movement. Before embodying the robot, the student and his peers had unsuccessfully tested their code to make the robot navigate the triangle's path. The robot navigated one side of the triangle and rolled away from the path without completing the task. In an attempt to debug and refine their code, the student decided to walk on the triangle path with his right hand stretched in front (see Figure 3a), holding the iPad with his left hand, adjusting directions on the virtual protractor as he walked on the taped triangle (see Figure 3b).

Figure 3

Student Maps the Triangle First Using His Right Hand (a), and Then Starts to Walk Around the Triangle While Looking at the Virtual Protractor (b)



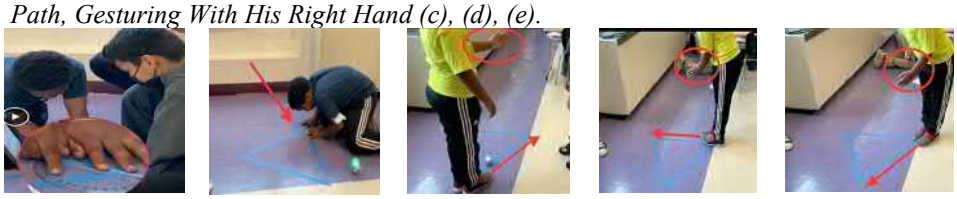
(a) (b)

The student's decision to embody both the robot and the path into his cognitive schema demonstrates the student's reliance on embodiment and gesturing as tools to support him refine his reasoning and ideas about mathematical concepts. Further, in reasoning about how the robot can move or change direction, he connects his spatial awareness and kinesthetic sense with the robot's path. By walking along the triangle's path, the student does not explicitly apply mathematical formulas or calculations to determine the angles. Instead, he uses his body along with the virtual protractor on the iPad to facilitate mapping the triangle's directions and angles, using the virtual protractor as a support tool from which he figured out the concrete values of directions. This approach provides insights into the learner's spatial awareness and ability to engage his body to abstract angles and directions by aligning them with the protractor's readings.

The iterative process of adjusting the angles using the virtual protractor and his body further shows that the student integrates his physical orientation as part of the tools he uses to problem solve, consequently providing us with cues that the learner makes connections between his body, gestures, interactions with the virtual protractor, and geometric reasoning. By continuously modifying the angles based on his body's alignment and the protractor's guidance, he provides evidence of an embodied way of reasoning about geometric relationships that blends physical interaction and digital tools to make abstract concepts more accessible. This example shows how students integrate physical actions with digital tools to reason about angles. This blends motor simulation (walking path) with perceptual input (protractor reading), aligning with GSA's idea that gesture and action can support cognitive processing.

Figure 4

(a) and (b) Student Measuring Angles with a Physical Protractor. Student Walking on a Triangle Path, Gesturing With His Right Hand (c), (d), (e).



(a) (b) (c) (d) (e)

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Measuring and simulating angles with tools and gestures

The third example is from clips from two consecutive lessons on programming triangles. We observed a third instance where the student used a physical protractor to measure the angles of the triangle, as seen in Figure 4 (a) and (b). The learners used a physical protractor to measure the 60-degree inner angles of the equilateral triangle and then entered values of the angles into their programs for the Sphero to navigate. The 60-degree angles entered did not make the robot roll as desired. After several tries, the robot kept rolling off the triangle's path. Sphero robots are designed to use a geocentric reference system rather than an egocentric system. Geocentric systems use a global reference frame based on the external environment, like the GPS or compass. Sphero robots are designed to move forward (0 degrees), move right (90 degrees), etc, using an internally designed compass system. For example, if the first angle the robot has to turn is 60, which is the case for an equilateral triangle, the next angle needs to be 180, i.e. (60 + 120). The learner grappled with understanding the geometric angles by referencing the robot's start position as scaffolds. After making measurements with a physical and virtual protractor, the student walked on the triangle's path with his right hand stretched, this time without the virtual or physical protractors (see Figure 5 (a), (b), (c)).

The student used his body and gestures to map the path of the equilateral triangle into his cognitive schema. The student assumed the robot operated using an egocentric system, where navigation and perception are based on the robot's local frame of reference. Without formal knowledge of such systems, the student relied on his own physical movements to align and ground his understanding of the robot's coordinate system. To program the Sphero robot to navigate an equilateral triangle, the vertice (angle) of 60 degrees for each vertice (angle) must be translated into the robot's "direction" (angle) on the roll block. For instance, the robot must make three 120-degree left turns, or 240-degree right turns at the vertices to follow an equilateral triangle path. These adjustments in understanding angles in dynamic activities with moving objects significantly shift from the static 60-degree angles typically taught in pen-and-paper contexts, presenting a challenge for young learners. The learner's spatial reasoning skills displayed in this activity were visible through his use of depictive dynamic gestures, through his use of hand gestures, and walking on the triangle's path to simulate the robot movement. The learner's consistent use of depictive dynamic gestures in this activity suggests that he engaged in action-based thinking as part of his cognitive processes, using embodied gestures to support his reasoning about the triangle's spatial properties in ways that suggest his geometric (mathematical) reasoning of the path and the angles. This example illustrates a shift from external measurement to internalized, embodied reasoning, providing evidence of how physical interaction and repeated gesture use can help internalize geometric concepts, supporting learning through embodied simulation.

Discussions

Our findings indicate that the focal student's thinking, reasoning, and learning were deeply intertwined with their embodied gestures and manipulations of tools in the environment. We found that the student used embodied gestures to reason about the geometric shape of a triangle. In all of our selected observations, the learner consistently used dynamic depictive gestures in three instances: swerving his palm and body to depict the turns(angle) the robot would make, walking on the triangle path, making adjustments on the virtual protractor on the iPad, measuring the triangle's angles using a physical protractor, and walking on the triangle with his right hand stretched out in front, without the virtual or physical protractor. Examining these gestures and embodied actions supports us in making inferences about what is happening in terms of geometric reasoning (Hostetter & Alibali, 2019).

The dynamic object (the robot) incorporated into the activity provoked the affordance of emergent dynamic gesturing by the learner. In this kind of activity, where learners are tasked with programming a Sphero robot to navigate a triangular path, learners find opportunities to walk on the path, embodying both the robot and the geometric shape of the path within their cognitive schema. Through gestures, such as hand movements and body swerves, to depict directions and angles, learners engage in embodied, emergent experiences that deepen their understanding of geometric shapes (Nathan & Walkington, 2017). Our observations revealed that learners without prior knowledge of the Sphero robot's internal coordinate system or the geometric properties of an equilateral triangle relied on embodiment and gesturing reasoning to interpret logical turns (angles) and directions the robot will travel on the triangle (Alibali & Nathan, 2012; Nathan et al., 2021).

Integrated CT and mathematics activities involving programming robots to navigate geometric paths require learners to recognize and apply computational and theoretical mathematics ideas in practical ways. This entails understanding and bridging the gap between abstract geometric concepts in theoretical mathematics and their practical application in problem-solving activities. This highlights the power of integrating CT into mathematics lessons to build a conceptual understanding of how young learners reason about complex mathematics concepts like angles when they program robots to navigate geometric shapes.

CT robotics activities integrated into mathematics lessons integrate a wide variety of tools and scaffolds that provide opportunities for young learners without programming or advanced logical-mathematical reasoning to use gestures and embodiment to support computational practices, complex cognitive interactions, and engage in mathematical reasoning that can be observable through grounded embodied cognition lens.

Like all studies, this work has its limitations. We focused our observations on just one student in a focal group, to understand how he used embodied gestures to reason about geometric shapes (equilateral triangle). Not all learners may have used embodied gestures to reason about shapes and angles as they programmed their robots to navigate geometric paths. We, therefore, cannot generalize or apply certain learning claims about young learners' use of embodied gestures to reason about geometry in robotics programming tasks. Studies with larger sample sizes could offer valuable insights into the role of embodied gestures in understanding geometric reasoning.

Conclusion

Examining complex contexts in which participants learn mathematics concepts like geometry in CT robotics contexts requires attending to learners' gestures and embodied actions. While geometric angles are introduced as part of the upper elementary mathematics curriculum lessons, they present significant complexity once introduced into robotics programming activities. We presented instances where participants used gestures and embodiment to reason about geometric shapes (triangles). Our work lays the foundation for further research into the role of embodiment and gestures in integrated mathematics, and CT-enhanced robotics tasks. Specifically, we aim to build on our findings to explore the discursive nature of learning with CT tools, such as robots, particularly when integrated into mathematics lessons. Our findings suggest that teachers can integrate CT and math by using robotics tasks that involve movement, gesture, and physical tools. Encouraging students to act out paths and spot patterns supports both spatial reasoning and algorithmic thinking, making abstract concepts more concrete.

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